A Model-Based Localized Damage Detection in Structural Health Monitoring

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ABSTRACT

A structural damage detection method is presented, labeled as Partitioned Damage Detection (PDD) method, which employs the recently developed displacement-only partitioned (DP) equations. The proposed PDD method starts with experimentally or simulation-generated modes and mode shapes. Second, it transforms the conventional assembled flexibility into DP-based partitioned flexibility. Third, the free-free forms of substructural flexibility is extracted from the DP-based flexibility. Fourth, Damage Indicator (DI) for each substructures are computed by the ratio of the traces of the *free-free* healthy and damaged substructural flexibilities. Numerical simulation and experiments show that the proposed PDD method correctly identifies both the damage location and damage levels for determinate structures. For indeterminate structures, the proposed PDD method correctly identifies the damage locations, while underestimating damage levels. A compensation scheme is developed for improving damage levels, which results in a reliable estimation of both damage locations and damage levels.

INTRODUCTION

The present study is focused on a new model-based method, labeled as *Partitioned Damage Detection* (**PDD**) method, with which one may be able to assess damage locations and, in particular, damage levels.

Our motivations for developing the proposed PDD method are as follows:

- Nearly all of the modern infrastructures, power plants, modern transportation vehicles (car, train, boats, airplanes, etc.) utilize Finite Element Method (FEM) for their design, performance evaluation and onsite-test correlations.

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- However, FEM models utilize **assembly** of many finite elements into the global equations. In so doing, while a FEM model yields the overall behavior of the model, they often mask its elemental behaviors.
- Structural health monitoring should offer not only damage state but more importantly damage locations (substructures) and their damage levels compared with their healthy states. This calls for a method that can, for each subsystems or partitions, individually assess its health state and/or damage levels.
- Recently, new FEM formulations have been developed for FEM modeling and solving that **do not require assembly** of elements or substructures, labeled as DP(Displacement-only Partitioned) formulation [1] and its dual called PartFlex method [2]. These new formulations yield partitioned flexibility from simulated or measured modes and mode shapes.
- In the present study, employing the partitioned flexibility, we will develop a method not only for detecting damage locations (in which elements or which substructures damage occur) but more importantly for assessing damage levels, employing the DP and PartFlex formulations

PARTSTIFF AND PARTFLEX FORMULATIONS

We recall from [1, 2] the coupled yet unassembled equations of motion as

PartFlex Equations [2]	PartStiff Equations [1]
$\mathbf{d} = \mathbf{F}_{pf}(\mathbf{f} - \mathbf{M}\ddot{\mathbf{d}})$ $\Downarrow$ $\mathbf{F}_{pf}\ddot{\mathbf{q}} + \mathbf{M}^{-1}\mathbf{q} = \mathbf{F}_{pf}\,\mathbf{f},\ \ \mathbf{q} = \mathbf{M}\mathbf{d}$	$\ddot{\mathbf{d}} = \mathbf{P}_s(\mathbf{f} - \mathbf{K}\mathbf{d})$ $\Downarrow$ $\mathbf{M}\ddot{\mathbf{d}} + \mathbf{K}_{ps}\mathbf{d} = \mathbf{P}_d\mathbf{f},\ \ \mathbf{K}_{ps} = \mathbf{P}_d\mathbf{K}, \quad \mathbf{P}_d = \mathbf{M}\mathbf{P}_s$

where the partitioned displacement, applied force, mass and stiffness matrices are given by

$$\mathbf{d} = \begin{Bmatrix} \mathbf{d}^1 \\ \vdots \\ \mathbf{d}^{N_s} \end{Bmatrix}, \quad \mathbf{f} = \begin{Bmatrix} \mathbf{f}^1 \\ \vdots \\ \mathbf{f}^{N_s} \end{Bmatrix}, \quad \mathbf{M} = \begin{bmatrix} \mathbf{M}^1 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{M}^{N_s} \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} \mathbf{K}^1 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{K}^{N_s} \end{bmatrix}$$

and the partitioned flexibility and partitioned stiffness matrices are given by

$$\begin{aligned} \mathbf{F}_{pf} &= \mathbf{F} - \mathbf{F}\mathbf{B}\mathbf{C}_{pf}\mathbf{B}^T\mathbf{F},\ \ \mathbf{C}_{pf} = \mathbf{P}_\lambda\mathbf{K}_b & \qquad \mathbf{P}_s &= [\mathbf{M}^{-1} - \mathbf{M}^{-1}\mathbf{B}\mathbf{C}_m\mathbf{B}^T\mathbf{M}^{-1}] \\ \mathbf{P}_\lambda &= \mathbf{I}_b - \mathbf{K}_b\mathbf{L}_f\,[\mathbf{L}_f^T\mathbf{K}_b\mathbf{L}_f]^{-1}\,\mathbf{L}_f^T & \qquad \mathbf{P}_m &= \mathbf{I}_b - \mathbf{M}_b\mathbf{L}_f\,[\mathbf{L}_f^T\mathbf{M}_b\mathbf{L}_f]^{-1}\,\mathbf{L}_f^T \\ \mathbf{K}_b &= [\mathbf{F}_b]^{-1}, \quad \mathbf{F}_b = [\mathbf{B}^T\mathbf{K}^{-1}\mathbf{B}] & \qquad \mathbf{M}_b &= [\mathbf{B}^T\mathbf{M}^{-1}\mathbf{B}]^{-1}, \mathbf{C}_m = \mathbf{P}_m\mathbf{M}_b \end{aligned}$$

$$\mathbf{F} = \begin{bmatrix} \mathbf{K}^1 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{K}^{N_s} \end{bmatrix}^+$$

Figure 1 schematically illustrates the essential difference between the assembled FEM equations and the PartStiff equations. Note that, whereas the conventional assembled FEM equations couples among the adjacent elements, the PartStiff method preserves that unassembled stiffness matrices as unassembled block-diagonal matrices. The necessary coupling among the partitioned (uncoupled) elements or substructures are accomplished by the coupling projector ($\mathbf{P}_d$).

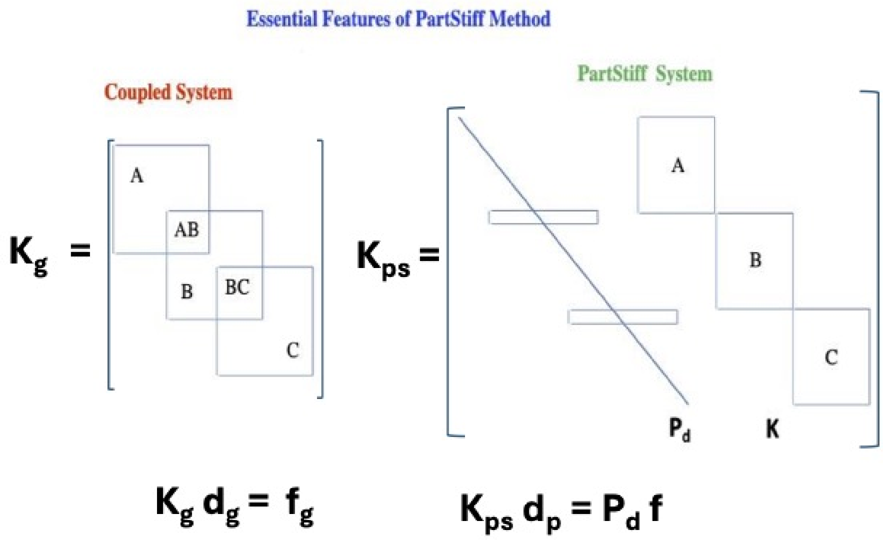

Figure 1. Matrix Profiles of Conventional Assembled vs. PartStiff FEM Equations.

DAMAGE INDICATOR OF PROPOSED PDD METHOD

It has been shown in previous studies [1, 2, 4] that the nonzero eigenvalues and eigenvectors of the two FEM equations are related according to

$$\begin{array}{lll} & \text{Assembled Equation} & \text{PartStiff Equations} \\ \text{Eigensystem:} & \mathbf{K}_g\mathbf{\Phi}_g = \mathbf{M}_g\mathbf{\Phi}_g\mathbf{\Lambda}_g & \mathbf{K}_{ps}\mathbf{\Phi}_{ps} = \mathbf{M}\mathbf{\Phi}_{ps}\mathbf{\Lambda}_{ps} \end{array}$$

$$\begin{array}{rl} \text{Nonzero Eigenvalues:} & \mathbf{\Lambda}_{ps} = \mathbf{\Lambda}_g \\ \text{Nonzero Eigenvectors:} & \mathbf{\Phi}_{ps} = \mathbf{L}_g\mathbf{\Phi}_g \\ \text{Displacement Relation:} & \mathbf{d}_{ps} = \mathbf{L}_g\mathbf{d}_g \end{array} \tag{1}$$

The intrinsic system energy associated with the vibrating system may be expressed as (see, e.g., [4]):

$$\begin{array}{ccc} \text{Energy} & \text{Assembled Equation} & \text{PartStiff Equations} \\ & E_g = \sum_{k=1}^{n_g} \frac{1}{(\omega_k^2)_g} & E_{ps} = \sum_{k=1}^{n_g} \frac{1}{(\omega_k^2)_{ps}} \\ & \Downarrow . & \Downarrow \\ & E_g = trace(\mathbf{K}_g^{-1}\mathbf{M}_g) & E_{ps} = trace(\mathbf{K}_{ps}^{+}\mathbf{M}_{ps}) \end{array} \tag{2}$$

Employing the preceding energy relations, we propose the present PDD (partitioned damage detection) criterion as follows:

$$\begin{array}{ccc} \text{Damage indicator (DI)} & \text{Assembled Equation} & \text{PartStiff Equations} \\ & DI_g = \frac{(E_g)_{healthy}}{(E_g)_{damaged}} & DI_{ps} = \frac{(E_{ps})_{heathy}}{(E_{ps})_{damaged}} \end{array} \tag{3}$$

In practice, however, it is not possible to measure all the modes from system identification. Moreover, identification of damage locations. To this end, we offer alternative DI formula for

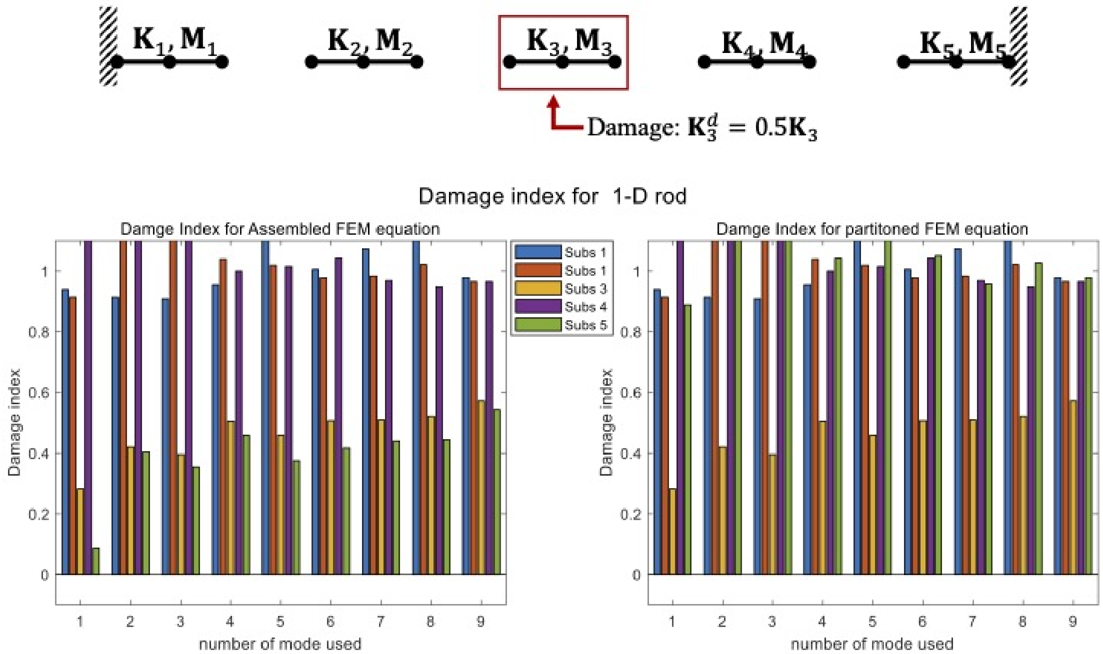

Figure 2. Fixed-Fixed Rod Problem

partitioned system as follows (for 3-partition system):

$$
\begin{aligned}
\mathbf{F}_{ps} &= \mathbf{L}_g \mathbf{\Phi}_g \mathbf{\Lambda}_g^{-1} \mathbf{\Phi}_g^T \mathbf{L}_g^T \\
&= \begin{bmatrix} \mathbf{F}_{ps}(1,1) & \mathbf{F}_{ps}(1,2) & \mathbf{F}_{ps}(1,3) \\ \mathbf{F}_{ps}(2,1) & \mathbf{F}_{ps}(2,2) & \mathbf{F}_{ps}(2,3) \\ \mathbf{F}_{ps}(3,1) & \mathbf{F}_{ps}(3,2) & \mathbf{F}_{ps}(3,3) \end{bmatrix} \\
\text{DI for partition i} &= \frac{trace(\mathbf{F}_{ps}(i,i))_{healthy}}{trace(\mathbf{F}_{ps}(i,i)))_{damaged}} \quad \text{(if fixed partition)} \\
&= \mathbf{P}_r [\frac{trace(\mathbf{F}_{ps}(i,i))_{healthy}}{trace(\mathbf{F}_{ps}(i,i)))_{damaged}}] \mathbf{P}_r \quad \text{(if free-free partition)}
\end{aligned} \tag{4}
$$

where $\mathbf{P}_r$ is a floating mode projector.

It turns out, when the entire modes are fully identified, the preceding formula may be replaced by the Schur complement, i.e.,

$$
\begin{aligned}
[\mathbf{F}_{ps}(i,i)]^S &= \mathbf{F}_{ps}(i,i) - \mathbf{F}_{ps}(i,k)[\mathbf{F}_{ps}(k,k)]^{-1}\mathbf{F}_{ps}(k,i) \\
&\Downarrow \\
\text{DI for partition (i)} &= \frac{trace[\mathbf{F}_{ps}(i,i)]^S_{healthy}}{trace[\mathbf{F}_{ps}(i,i)]^S_{damaged}}
\end{aligned} \tag{5}
$$

PERFORMANCE OF PROPOSED PDD-BASED DAMAGE INDICATOR

In order to evaluate the effectiveness of the proposed Damage Indicator (DI) formulations (4) and (5), a simple fixed-fixed 1D rod problem is selected, as shown in Figure 2. The rod is divided into five substructures, each consisting of two rod elements. Despite its simplicity, this example effectively demonstrates three key attributes: it is an indeterminate structure, damage is localized in partition 3 with stiffness reduced to 50%, and it illustrates the impact of using a reduced number of identified modes.

Ideally, in the undamaged state, the DI values for substructures (1, 2, 4, 5) should be 1, while the DI value for the damaged partition (3) should be 0.5. Figure 2 illustrates the variations in DI values based on the number of modes considered. When the assembled FEM equations are applied, the DI value for partition 5 erroneously converges to 0.54 not 1. In contrast, when the proposed PartStiff equations are employed, even with a limited number of modes, the method correctly identifies the damaged partition (3). Additionally, as the number of modes increases, the DI value for partition 3 converges to 0.5, further validating the effectiveness of the PartStiff method in accurately detecting both the damage location and extent.

The proposed DI method has been further evaluated using numerical experiments involving beams and other structural configurations, yielding promising results. Current efforts are focused on developing DI digital twins capable of real-time structural health monitoring. Further findings and methodological advancements will be discussed in the conference presentation. It is noted that the PartStiff and its allied PartFlex methods have been applied to number of important computational mechanics problems [1]- [7], some of which pertinent to structural health monitoring will also be presented at the workshop.

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