Monitoring of Civil Engineering Structures Based on H_∞ Estimation

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ABSTRACT

Subspace methods, a field of numerical mathematics, are applied to system identification of mechanical structures. The efficacy of these methods, applicable to both deterministic and stochastic model estimation, has been proven on numerous occasions in practical and research applications. The system identification process yields multidimensional state space systems, which serve as the mathematical foundation for the models. In the context of stochastic system identification for mechanical systems, particularly those exposed to ambient noise sources such as wind and traffic, output-only methods are employed. Since the system inputs are unknown, the model can't be completely determined. By applying $H\infty$ optimization methods, models with a theoretical normalized input are obtained. These state space systems can be interpreted as a digital twin at a designated time point.

In this contribution the method "Autonomous Model Order Selection" (AMOS) is linked with the structural health monitoring (SHM) method "State Projection Estimation Error" (SP2E), which leads to an automatic and continuous SHM approach. The output of the SHM method is a damage localization indicator, which is combinable to a digital twin. This approach is verified with laboratory measurements and a large-scale experiment on a real bridge.

INTRODUCTION

The condition of mechanical structures deteriorates over time. To extend the lifetime of civil engineering structures, Structural Health Monitoring (SHM) became an active field of research. Early detection of damage enables timely and cost-effective repairs, which not only extends structural lifespan but also enhances safety. Detecting

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damage at an early stage allows the closure of compromised structures ahead of time, potentially saving lives.

To be practical across a large number of structures, SHM must provide an automatic and periodic assessment of structural condition. This paper presents an SHM approach that achieves this using the State Projection Estimation Error (SP2E) method [1,2], which is based on output-only system identification. At defined time intervals, state-space models are identified using covariance-driven Stochastic Subspace Identification (SSI-cov) – a black-box technique that linearizes the system dynamics at an operational point.

However, structural behavior varies over time, and Environmental and Operational Conditions (EOC) significantly influence system dynamics. Differentiating between changes caused by damage and those induced by varying EOC is therefore essential.

This paper first outlines the theoretical foundation of the proposed SHM framework. It then presents experimental validation, starting with controlled laboratory tests and concluding with a large-scale field study on the Flossgraben Bridge, where EOC vary naturally.

PROPOSED STRUCTURAL HEALTH MONITORING FRAMEWORK

In this section, the framework for automatic monitoring at regular intervals is presented, using the SHM method SP2E, which is based on identified and H_∞-optimized black-box state space systems.

System Identification

Since the goal is to provide an automated SHM approach, the stochastic realization approach based on the SSI algorithm is used to obtain black-box state-space systems. The input for the method is the multidimensional covariance function $R_y(\tau)$, which is computed by the measured output y_k (E denotes the expectation value).

$$R_y(\tau) = E\{y_k y_{k+\tau}^T\} \tag{1}$$

The sought parameters are obtained by performing a singular value decomposition (SVD) on the block Hankel matrix $\mathcal{H}$, where the multidimensional covariance function is arranged with k blocks [3].

$$\mathcal{H} = \begin{bmatrix} R_y(1) & R_y(2) & \dots & R_y(k) \\ R_y(2) & R_y(3) & \dots & R_y(k+1) \\ \vdots & \vdots & \ddots & \vdots \\ R_y(k) & R_y(k+1) & \dots & R_y(2k-1) \end{bmatrix} = [U_1 \quad U_2] \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} \tag{2}$$

By only considering the significant singular values Σ_1 and the corresponding vectors U_1 and V_1, the parameters A, B, C and D, describing the identified state space system $T\{A, B, C, D\}$, are estimated. During the identification process, the question of the size of the immeasurable state space n arises, which is equal to the dimension of the significant singular values Σ_1. Only the significant singular values are taken into account

to separate the signal space from the noise space. The noise space arises due to measurement noise and numerical inaccuracies during the identification process.
Classically, this issue is addressed by constructing a stabilization diagram, in which the modal characteristics such as frequency and damping of the identified systems are analyzed. However, a drawback of the stabilization diagram is that it requires human interaction to select a model order n. To automate the identification process, the Autonomous Model Order Selection (AMOS) method was developed. AMOS is based on the energy of the system states. Although there are approaches that automate the stabilization diagram using machine learning algorithms (e.g., see [4,5]), SP2E relies on the difference in average process power of identified state-space systems. Therefore, AMOS uses the energy error ε to select the model order that best represents the measurement and is thus most suitable for the SP2E method.

$$\varepsilon_n = \sum_{i=1}^{n} \sigma_i(\mathcal{H}) - \sum_{i=1}^{n} \left|\lambda_i\left(W_{x,0,k-1}\right)\right| \tag{3}$$

Equation (3) shows the calculation of the energy error for a specific model order n, given by the difference between the "measured energy" $\sigma_i(\mathcal{H})$ and the "identified energy" $\left|\lambda_i\left(W_{x,0,k-1}\right)\right|$. The "measured energy" results from the sum of the used singular values of the block Hankel matrix, while the "identified energy" is computed from the estimated parameters via the time-limited cross Gramian. For details on the energy determination, see [6]. After computing the energy error for each model order within a predefined search interval – which serves as a coarse separation between signal and noise space – the model order with the minimum energy error is selected as the best representation of the measurement.

H_∞ optimization

The measurements from which the state-space systems are identified represent the structural behavior at a specific point in time. Since the structures are monitored during operation, the input power varies between measurements. Consequently, the average process power of the identified systems also differs. To ensure comparability between the identified state space systems, an H_∞ optimization is applied, resulting in the input process normalized state space systems described by the parameters A, K_p, C and I. The power spectrum of these normalized state space systems is expressed through a product decomposition. During the optimization process, a Riccati equation (see Equation (4)) must be solved to compute the gain matrix K_p (see Equation (5)).

$$P = APA^T + I - AP\begin{bmatrix} C^T & L^T \end{bmatrix}\left(\begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} CPC^T & CPL^T \\ LPC^T & LPL^T \end{bmatrix}\right)^{-1}\begin{bmatrix} C \\ L \end{bmatrix} PA^T \tag{4}$$

$$K_p = APC^T(CPC^T + I)^{-1} \tag{5}$$

The matrix P, which is obtained by solving the Riccati equation, represents the estimated state error covariance matrix. The optimization case "filtering in additive noise" is applied; therefore, $L = C$ is chosen. To obtain the pure estimator, $\gamma = 1$ is set, which is known as the upper bound [7].

State Projection Estimation Error

The input process normalized state space systems introduced in the previous subsection build the basis for the SHM method SP2E. For monitoring purposes, three systems are interconnected. The input signal is first processed by a normalization model T_0. The output of this model is then simultaneously fed into two parallel systems: the reference model T_i and the monitoring model T_j. These models represent the baseline (undamaged) system and the system under current observation (potentially damaged or altered), respectively. The outputs of these systems are subsequently processed by a difference process, which computes the deviation between the two model responses.

$$\left(T_j^{-1} - T_i^{-1}\right)T_0 = \begin{bmatrix} A_j - K_{p,j}C_j & 0 & 0 & K_{p,j} \\ 0 & A_i - K_{p,i}C_i & 0 & K_{p,i} \\ 0 & 0 & A_0 & K_{p,0} \\ C_j & -C_i & 0 & 0 \end{bmatrix} = \begin{bmatrix} \tilde{A} & \tilde{B} \\ \tilde{C} & \tilde{D} \end{bmatrix} \quad (6)$$

The interconnected system (see Equation (6)) is evaluated by computing the state covariance $\tilde{\Pi}$, followed by a computation of the average process power, which is the damage localization indicator P_{dV}.

$$\tilde{\Pi} = \tilde{A}\tilde{\Pi}\tilde{A}^T + \tilde{B}\tilde{B}^T \quad (7)$$

$$P_{dV} = \mathrm{diag}(\tilde{C}P_V\tilde{\Pi}P_V^T\tilde{C}^T) \quad (8)$$

To eliminate inaccuracies caused by different EOC, the states of the normalization model are projected onto the reference model and the monitoring model via the projection matrix P_V.

$$\begin{bmatrix} A_i - K_{p,i}C_0 & 0 \\ 0 & A_j - K_{p,j}C_0 \end{bmatrix}\Theta - \Theta A_0 = -\begin{bmatrix} K_{p,i} \\ K_{p,j} \end{bmatrix}C_0 \qquad \Theta = \begin{bmatrix} \Theta_i \\ \Theta_j \end{bmatrix} \quad (9)$$

$$P_V = \begin{bmatrix} 0 & 0 & \Theta_i \\ 0 & 0 & \Theta_j \\ 0 & 0 & I \end{bmatrix} \quad (10)$$

Framework

The proposed framework connects system identification, input process normalization via the H_∞ optimization, and damage localization. As a result, the damage localization indicator P_{dV} is obtained automatically at defined time intervals. Figure 1 depicts the proposed framework.

The method SP2E consists of two phases: the learning phase and the monitoring phase. During the learning phase, the output of the monitoring structure is recorded over a defined time interval Δt (e.g. $\Delta t = 10min$). While the structure is being monitored, the previous measurement is processed. First, the output data is converted to a multidimensional covariance function and organized into a block Hankel matrix. Next, the model is computed automatically using SSI in combination with AMOS. The

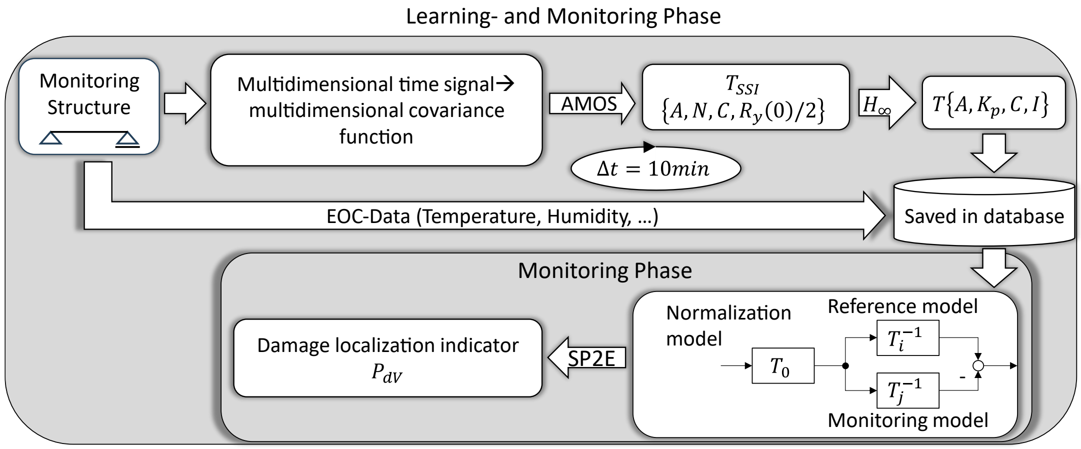

Figure 1: Framework of proposed SHM method.

resulting input process normalized state space system is stored in a database along with the measured EOC data. This procedure is repeated until a sufficient number of models representing different EOC have been stored. Once this is achieved, the learning phase concludes and the monitoring phase begins.

In the monitoring phase, new models of the structure are continually identified from output measurements. The most recent model becomes the current monitoring model T_j. A suitable reference model T_i is then selected using a pairing algorithm (e.g., k-Nearest-Neighbor) based on the recorded EOC data. This selection is essential, as comparing models from dissimilar EOC may mask damage or result in incorrect localization [2].

EXPERIMENTAL RESULTS

This section presents experimental results related to the presented damage localization framework. Due to visualization reasons, only exemplary results from one measurement are presented. Here, a damage is interpreted as a structural system change, typically caused by variation in stiffness or mass.

First, laboratory results are introduced, followed by a large-scale experiment conducted on the Flossgraben Bridge.

Acceleration data are collected using uniaxial piezoelectric sensors (PCB 393A03). A real-time modular measurement system is employed for data acquisition, supporting up to 72 acceleration channels with a sampling rate of 10 kHz per channel. Details on the measurement system and the laboratory structure are provided in [8].

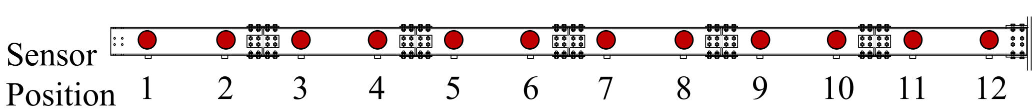

Figure 2: Technical drawing of the laboratory structure. The sensor positions are marked by circles. The clamping of the cantilever is at the right side.

Laboratory

LABORATORY STRUCTURE AND MEASUREMENT

The laboratory setup at I4S consists of a steel cantilever composed of six IPE200 profiles connected by screws. Each profile measures approximately one meter length, resulting in a total length of about six meters. The overall mass of the structure is 189 kg. Excitation is provided by three wind machines, positioned between sensor locations one and four. A technical drawing of the structure is shown in Figure 2.
Measurements are conducted over ten-minute intervals. To eliminate noise, the measurement data are low pass filtered with a cutoff frequency of 200 Hz.

LABORATORY EXPERIMENT AND RESULTS

As previously mentioned, a system change is caused by mass and/or stiffness alterations. Both types of alteration are demonstrated on the laboratory structure.
For mass alterations, several configurations were tested, including reference measurements (no added mass) and the placement of additional mass between sensor positions 1-2, 4-5, 7-8, and 10-11. The additional mass is 2.8 kg, corresponding to approximately 1.5 % of the total structural mass.
The alteration in stiffness is induced by removing all eight stay bolts located between sensor position 2 and 3. To ensure that only stiffness changes are detected – rather than mass changes – the removed mass from the bolts and web plates is compensated using magnets.
For automatic processing, the block Hankel matrix is constructed with $k = 200$ rows and columns. The AMOS method is applied to determine the optimal model order, using a search interval from $n_{min} = 24$ and $n_{max} = 100$.
Results are displayed in Figure 3. Due to visualization reasons, only one result per system change is shown. The evaluation of multiple measurements lead to robust,

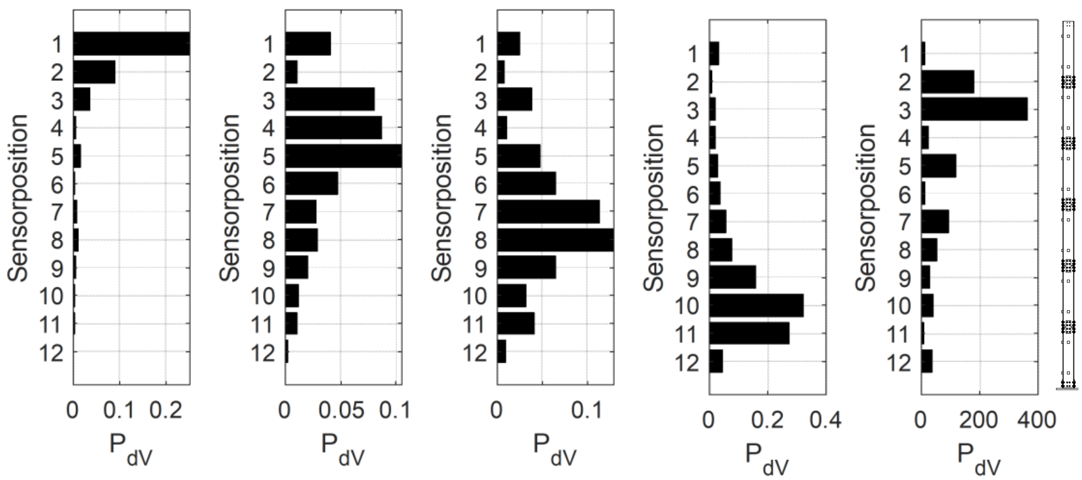

Figure 3: Damage localization indicator of different system changes. From left to right: mass perturbation between sensor positions 1-2; 4-5; 7-8; 10-11 and stiffness alteration between sensor positions 2-3.

comparable results. The y-axis represents sensor positions and the x-axis shows the damage localization indicator. High values on the x-axis indicate the presence of a system change. The system change is clearly seen for the stiffness alteration between sensor position 2 and 3 as well as the mass alterations between the different sensor positions. Thus, mass alterations are an adequate system change to simulate a reversable damage.

Large-Scale Experiment

FLOSSGRABEN BRIDGE

For the large-scale experiment, the Flossgraben Bridge was provided by the Landesstraßenbaubehörde Saxony-Anhalt [9]. The bridge, built in 2001, has a total length of 358 meters and features seven spans. Its superstructure consists of a steel box girder combined with an in-situ concrete slab, and the total structural mass is approximately 4750 tons. For this experiment, eight acceleration sensors were installed per span, resulting in a total of 56 acceleration sensors.

LARGE-SCALE EXPERIMENT AND RESULTS

The experiment was carried out from September 10th, 2024, to September 12th, 2024. As a reversible stiffness change under operational conditions was not feasible, a mass alteration was introduced by blocking one traffic lane and adding mass. The first day served as a reference, while the subsequent days involved mass alterations using cargo trucks. Initially, the trucks were positioned on span four, and on the following day, they were moved to span three. Figure 4 shows an aerial view of the bridge, including the trucks positioned at span 4.

The measurement data was low pass filtered with a cutoff frequency of 80 Hz to eliminate noise. The model order was selected using a search interval from $n_{min} = 10$ to $n_{max} = 15$. To reduce the impact of varying EOC, 50 measurements were averaged. The results are illustrated in Figure 5. Here, the x-axis indicates the sensor positions, where even numbers correspond to sensors on the eastern side of the bridge (gray) and odd numbers to those on the western side (black). The y-axis shows the values of the damage localization indicator.

The results clearly show that the position of the trucks is identified by the damage localization indicator.

Figure 4: Picture of the Flossgraben Bridge. Trucks (additional mass) positioned at span 4.

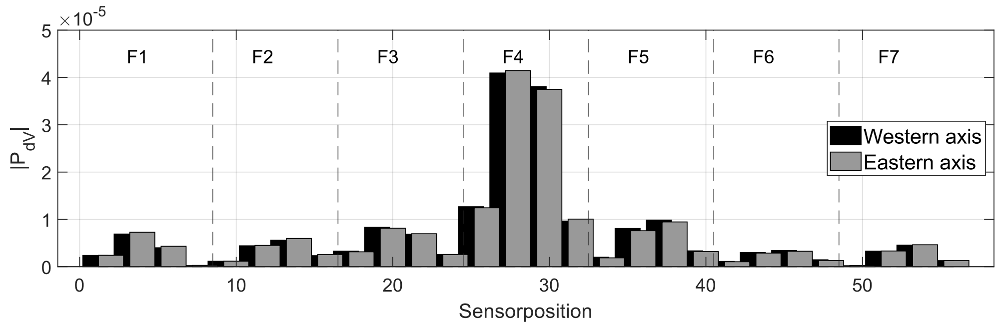

Figure 5: Damage localization indicator of mass alteration in span 4.

CONCLUSION

This paper presented a fully automated SHM approach based on output-only system identification and the SP2E method. The effectiveness of the method was demonstrated both in a controlled laboratory setup and in the large-scale experiment on the Flossgraben Bridge. It was also demonstrated that alterations in mass and stiffness are equivalent structural changes. The key output is a global damage localization indicator, which is suitable for implementation in a digital twin framework. The spatial resolution of the method is dependent on the number of sensors installed in the structure. It is important to note that the proposed method provides a preliminary assessment of the structural condition. If the damage localization indicator exceeds a defined threshold, a system change is inferred.

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