Full Waveform Inversion for High-Resolution Imaging and Defect Detection in Composites: On the Choice of Misfit, Regularization, and Parameterization

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ABSTRACT

Detecting defects in materials and obtaining high-resolution images of the materials' internal structures are essential for ensuring quality and structural integrity. Nondestructive testing (NDT) is favored for these purposes, thanks to its economic and environmental advantages. Among the various NDT techniques, ultrasound imaging shows great promise but entails several technical challenges, such as reliability and resolution. Full waveform inversion (FWI), an advanced optimization method, can generate high-resolution images of a material's internal structure and identify defects. In principle, FWI minimizes the misfit between recorded ultrasonic signals and simulated ones, often through iterative optimization methods. However, the fact that FWI is highly nonlinear often hinders the reliability of the reconstructed images. This work explores different strategies to improve FWI for both high-resolution imaging and defect detection, focusing on defects such as delamination in composites. More specifically, it emphasizes the choice of misfit measures, regularization methods, and parameterization techniques; studies their impact on FWI improvements; and provides suggestions based on the resulting findings. Various misfit measures—including the L_2-norm, L_1-norm, cross-correlation (CC), and envelope measures—are compared in order to assess their effectiveness in improving FWI outcomes. Furthermore, different regularization methods were investigated for stabilizing FWI and enhancing its accuracy. Simple regularization methods (e.g., Tikhonov regularization) were compared with more advanced techniques (e.g., total variation regularization). Also, different parameterization strategies and their impacts on FWI performance were examined. By applying model constraints through reparameterization techniques, such as using Sigmoid transformations of model parameters and projecting them to expected ranges, this work aims to improve reconstruction accuracy and convergence. These findings suggest that CC functionals augmented with total variation regularization, while also applying Sigmoid transformation to the model parameters, is a particularly effective strategy for FWI in multilayered systems/materials with sharp-edged defects (e.g., delamination in composites

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INTRODUCTION

Developing novel materials capable of withstanding extreme operating environments is of paramount interest in a variety of fields [1, 2]. Ensuring the quality and structural integrity of these materials necessitates advanced nondestructive testing (NDT) methods for detecting defects and obtaining high-resolution images of the materials' internal structures. Among these various NDT techniques, laser ultrasonics (LU) [3] is ideally suited for extreme environments. However, it relies on an analytical data processing method that assumes numerous simplifications and idealizations, thus limiting its effectiveness at characterizing complex materials and intricate geometries [3, 4]. Common ultrasonic reconstruction methods include the ray-based synthetic aperture focusing technique [5] and time of flight [6]. While ray-based methods work well for simpler structures, they are ineffective for complex ones (e.g., ones with complex geometries and/or impedance contrasts). On the other hand, wave-equation-based imaging techniques provide greater sensitivity for imaging, as they consider travel time, amplitude, and waveform distortions by using the solution of full acoustic or elastic wave equations. State-of-the-art wave equation imaging techniques employ full waveform inversion (FWI) to invert ultrasonic measurements [7]. Predominantly used in geophysical imaging, FWI is an advanced optimization method for generating high-resolution images of a material's internal structure and identifying defects by using iterative optimization to minimize the misfit between the recorded ultrasonic signals and the simulated ones. However, FWI is highly nonlinear and may be unreliable, especially in the presence of noisy and/or sparse measurements [8].

Recent advancements in regularizing FWI have shown promise for improving the stability and accuracy of the inversion process. Regularization techniques play a crucial role in mitigating the ill-posed nature of FWI, where small errors in the data can lead to significant errors in the reconstructed model. Among these techniques, Tikhonov regularization [9] has seen widespread use due to its simplicity and effectiveness in imposing smoothness constraints on the solution. However, more advanced methods (e.g., total variation regularization [10]) have been gaining attention for their ability to preserve sharp interfaces and discontinuities in the material properties. Additionally, novel approaches such as sparsity-promoting regularization [11] have been explored to enhance the imaging of sharp edges by encouraging sparse representations of the model parameters. The present work explores the application of these techniques to enhance FWI in the context of material characterization with LU, and compares the performance of these regularization methods within the FWI framework, the goal being to identify the optimal approaches for achieving reliable, high-resolution material characterization in conjunction with LU techniques.

This paper also explores different strategies to improve FWI of LU by optimizing the choice of misfit measures and parameterization methods. Various misfit measures—including the L_2-norm [12, 13], L_1-norm [11], cross-correlation (CC) [14, 15], and envelope measures [16]—were assessed in regard to their effectiveness in the context of applying FWI to LU data. Moreover, different parameterization strategies, including reparameterization techniques such as Sigmoid transformations of model parameters, were examined to enhance reconstruction accuracy and convergence [17, 18]. By comprehensively addressing these aspects, this work aims to develop a robust and efficient FWI-based reconstruction method for detecting defects in materials.

The remainder of the paper is organized as follows. The next section briefly describes the methods used and covers several different approaches for enhancing the efficiency of FWI. This is followed by some concluding remarks.

METHODS

LU is a non-contact NDT technique in which laser pulses generate and detect ultrasonic waves within materials. This makes it ideal for harsh environments. FWI is a nonlinear optimization technique that can reconstruct high-resolution images of material properties by minimizing the difference between the measured data (observed data, $\boldsymbol{v}_{obs}$) and the simulated data $\boldsymbol{v}_{sim}$, where $\boldsymbol{v}_{sim}$ is the wave equation solution:

$$\rho \partial^2 \boldsymbol{v}_{sim} / \partial t^2 = \nabla \cdot (\boldsymbol{E} \, \nabla \boldsymbol{v}_{sim}) + f \tag{1}$$

where ρ is the material density, t is the time, ∇ is the gradient operator, $\boldsymbol{E}$ is the material elastic tensor (often the unknown to be reconstructed), and f is the source term representing the laser impulse that induces mechanical waves. The difference between $\boldsymbol{v}_{obs}$ and $\boldsymbol{v}_{sim}$ is computed through a misfit function (e.g., $J(\boldsymbol{m}) = \|\boldsymbol{v}_{sim} - \boldsymbol{v}_{obs}\|_p$, where $\boldsymbol{m}$ could be ρ, $\boldsymbol{E}$, or any other material property of interest, and p could be 1 for the L_1-norm, 2 for the L_2-norm, or any other misfit measure we select). The imaging problem is thus formulated to minimize $J(\boldsymbol{m})$:

$$\widehat{\boldsymbol{m}} = \underset{\boldsymbol{m} \in \mathcal{M}}{argmin} \, J(\boldsymbol{m}) \tag{2}$$

where $\mathcal{M}$ represents all possible solutions and $\widehat{\boldsymbol{m}}$ represents the optimal solution (the image). The standard FWI updates $\boldsymbol{m}$ via a gradient-based optimization method. Straightforward application of FWI to LU data often converges to incorrect images, for two main reasons:

- The imaging problem is usually ill-conditioned.
- FWI is highly nonlinear and dependent on an initial guess.

To address these two issues, we explored different choices of misfit measures, regularization methods, and parameterization techniques; studied their impact on FWI improvements; and provided suggestions based on our findings.

SETUP

For a sandwich composite with a delamination defect, we considered the reconstruction of the pressure wave speed C_p, as shown in Figure 1(a). The initial model used to start the inversion process is depicted in Figure 1(b). For synthetic data generation and simulation, laser ultrasonic pulses were replaced by point sources distributed over the top and bottom surfaces of the specimen. A total of 24 pulses (sources) and 125 sensors (receivers) were used on each surface. Figure 1(c) presents an example of the acquired signal over the top surface, as recorded over time for the

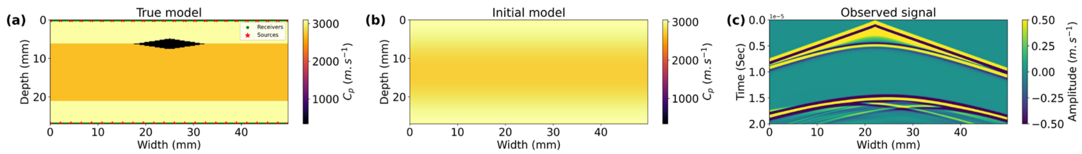

Figure 1. FWI setup: (a) true velocity model, (b) initial model, and (c) signal measured on the surface. Note that the surface wave is acquired over the top and bottom surfaces only.

given source. The signal used was a Ricker pulse with a central frequency of 0.5 MHz and a duration of 20 μs. We used the FDTD software Deepwave [18] to perform the ultrasonic wave simulation and inversion. The LBFGS optimization algorithm was used in all the experiments, with a maximum of 20 internal iterations and 30 outer ones. For the line search within the LBFGS, the Strong Wolfe algorithm was employed.

CHOICE OF MISFIT FUNCTION

Most FWI applications use the L_2-norm measure to compute the misfit function. However, these types of misfit functions exhibit highly nonlinear behavior with respect to the inversion parameters. To mitigate L_2-norm-misfit-related issues, other misfit functions have been proposed in the geophysical field. One example is the envelope function [16], which was derived from the analytic signal of the recorded waveform. The analytic signal is a complex-valued function whose real part is the original signal and whose imaginary part is the Hilbert transform of the original signal. The envelope of a signal provides a smooth curve that outlines its extremes.

Mathematically, if $s(t)$ is the original signal, the analytic signal $S(t)$ can then be:

$$S(t) = s(t) + j \cdot \boldsymbol{H}\big(s(t)\big) \tag{3}$$

where $\boldsymbol{H}(s(t)\,)$ is the Hilbert transform of $s(t)$, and j is the imaginary unit. The envelope $E(t)$ of the signal can then be obtained as the magnitude (or modulus) of the analytic signal:

$$E(t) = |S(t)| = \sqrt{s(t)^2 + \Big(\boldsymbol{H}\big(s(t)\big)\Big)^2} \tag{4}$$

where $s(t) = \boldsymbol{v}_{obs}$ when computing $E(t)_{obs}$ (envelop of the observed data), and $s(t) = \boldsymbol{v}_{sim}$ when computing $E(t)_{sim}$ (envelop of the simulated signal). Hence, the envelop-based misfit function becomes:

$$J(\boldsymbol{m}) = \|E(t)_{sim} - E(t)_{obs}\|_p \tag{5}$$

In the context of FWI, the envelope function is used to emphasize global changes in the amplitude of the waveforms, potentially helping mitigate cycle-skipping-related issues and improving the inversion result. Other misfit functions include the L_1-norm measure, which computes the absolute difference between the observed and simulated signals and is generally robust to outliers [11]. The CC misfit functions [14, 15] emphasize the correlation between the signals instead of the amplitude, relaxing the nonlinearity—especially at higher frequencies—and thus helping mitigate the issue of cycle skipping, which is a potential problem in most FWI frameworks [8].

In this work, we explored all four types of misfit functions for imaging material properties and detecting defects (delamination). Figure 2 demonstrates a comparison of these four types of misfits. As seen, none of the misfits fully reconstructed the shape of the defect. The CC misfit was the most accurate and exhibited fewer artifacts in the image, thus indicating robustness against the nonlinearity of the FWI. This necessitates the use of strong regularizers, which will be discussed in the next section.

CHOICE OF REGULARIZATION

In the previous section, we explored different types of misfit functions, revealing that the CC function might be the most robust for FWI in the context of material characterization with LU. In this section, we examine various regularization techniques to enhance defect shape reconstruction when using the CC function. Specifically, we explore Tikhonov regularization, L_1 regularization, and TV regularization. Tikhonov regularization stabilizes the solution by discouraging large parameter variations and is computationally simple to implement. However, it may over-smooth the solution, blurring sharp edges [9]. L_1 regularization promotes sparsity, making it effective for feature selection and handling noisy data while also preserving edges; however, its non-differentiability at zero can complicate optimization and introduce bias [11]. TV regularization preserves edges and reduces noise by discouraging large variations in smooth regions [10]. Figures 3 and 4 compare the three types of regularization. While some enhancement is observed, it demonstrates that none of these techniques could correctly reconstruct the delamination shape. Hence, further refinement of the FWI is needed, possibly by reparametrizing the FWI problem to enhance its sensitivity to such defects.

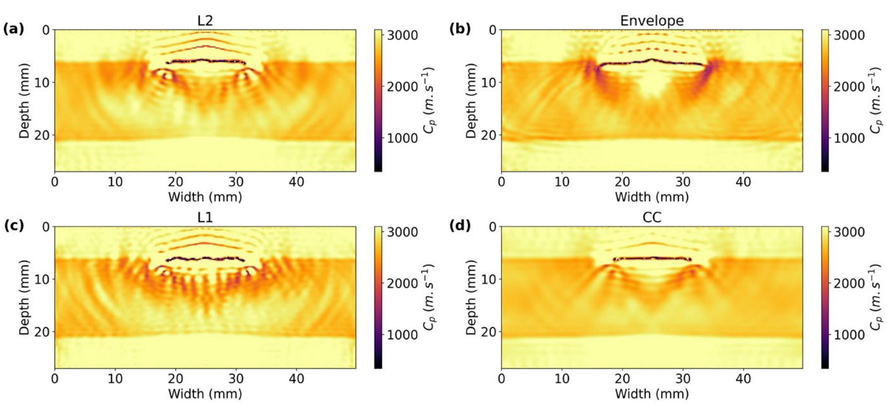

Figure 2. Comparison between different misfit functions: (a) L_2 misfit, (b) envelop-based misfit, (c)L_1-misfit, and (d) CC misfit.

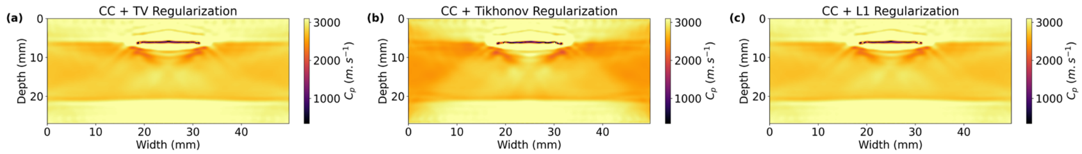

Figure 3. Comparison of different regularization techniques: (a) TV, (b) Tikhonov, and (c) L_1 regularization. The images are shown for iteration 5.

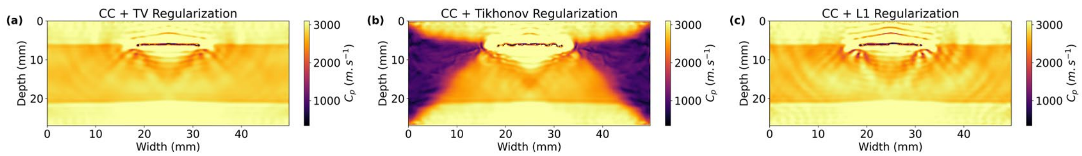

Figure 4. Comparison of different regularization techniques: (a) TV, (b) Tikhonov, and (c) L_1 regularization. The images are shown for iteration 30.

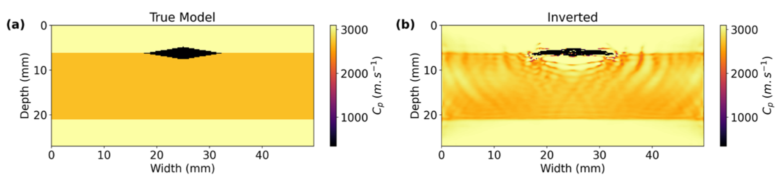

Figure 5. Final reconstructed image: (a) true velocity model and (b) reconstructed image when using the CC/TV and reparameterization strategy described in this section.

CHOICE OF PARAMETERIZATION

Next, we examined different parameterization strategies and their impact on FWI performance. Specifically, we applied model constraints through reparameterization by transforming the parameters from their original space to a Sigmoid space using Sigmoid transformations of the model and projecting them to expected ranges (e.g., the minimum and maximum of the expected velocity values). Figure 5 depicts the reconstructed image after applying this technique, in tandem with the CC misfit function and TV regularization.

DISCUSSION

Further refinement of the image could be achieved via a multi-resolution approach [12, 13, 15], with low-frequency content being used first, followed by the addition of higher frequencies as needed for detailed images. This approach essentially reduces the nonlinearity of the FWI and can result in better reconstruction.

Furthermore, in many cases—especially when the contrast in the material impedance is large—the gradient exhibits high values at particular locations (e.g., around sources and receivers and at sharp edges). These large gradients can affect the update and potentially lead to incorrect images. In such cases, clipping the gradient [18] may be a potential solution, as it ensures that the gradient remains well-behaved and that the update is within expected values.

CONCLUSIONS

This work explored various strategies to enhance FWI for high-resolution imaging and defect detection in composite materials, specifically targeting delamination. The key aspects investigated were misfit functions, regularization methods, and parameterization techniques. Of the four types of misfit measures, the CC functional proved the most robust and accurate, reducing the number of artifacts and mitigating nonlinearity and cycle skipping issues. For regularization, total variation regularization outperformed Tikhonov and L_1 regularization by better preserving sharp edges and providing better images with fewer artifacts. Reparameterization using Sigmoid transformations improved FWI performance by accurately reconstructing the delamination defect. Despite these advancements, some artifacts and inaccuracies in defect shape reconstruction remained, indicating the need for further refinement. Potential improvements could entail a multi-resolution approach and gradient clipping to better manage large gradients.

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