Forming an Advanced Proper Orthogonal Decomposition-Based Data Science Framework for Advanced Diagnostics of Practical Multi-Body Systems: SHM Challenges in Complex Structures-and-Flexible Machinery

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ABSTRACT

In an attempt to tackle science-challenging SHM issues in complex multi-body structural and machinery systems, we are developing a methodology that exploits the principles of continuum mechanics (theoretical-computational-experimental) along with optimum reduction analysis to lay data science aspects for data-driven mechanics. The focus is on damage diagnostics in the presence of structural complexity: domain geometry and nonlinearity distribution. Simultaneous ensembles of accelerations form basic datasets to be augmented properly. We introduce two fundamental augmentations of a dataset. The first, referred to as the Simultaneous Three-Point Observation (S3PO), is the augmentation of a dataset with another two similar datasets. The second, referred to as the Spatial Differential Form (SDF), is the augmentation of a dataset with its spatial differential forms of order-1,2. The APOD resolution interpretation of the S3PO datasets of composite beam structures reveals coherence among the individual datasets forming the augmentation: they intersect along modal physics-carrying POD modes. The diametrically opposite is revealed for the rotor structure, modified with holes: the datasets do not intersect at all. This is clearly due to the fact that the dynamics are infested with a great degree of uncertainty due to the phenomenon of chaotic wave scattering. The APOD resolution of the SDF dataset of composite beams extracts the actual bending curvature distribution, indicating directly the structural health state footprint imposed by the thermodynamical manufacturing process. The introduced data augmentation operations and the physics extraction by the APOD reduction analysis constitute the basis for a sound data science in engineering mechanics-physics with impact on SHM.

INTRODUCTION

Structural health monitoring concepts that are economically feasible and technologically feasible are indispensable for maintaining the characteristics of autonomy and mobility in physical transport infrastructure and beyond. The complexity of the geometry of the integrated transportation system (multi-body structure with flexible machinery) along with the plethora of local joint nonlinearities challenge reliable thorough damage diagnosis and exploitation of impulse-induced interrogation acceleration signals [1]. This challenge can potentially be met by unconstrained order reduction processing of experimental dynamics datasets in complex multi-body structural systems. The Proper Orthogonal Decompositions (POD) method offers an unparalleled dimension-reduction analysis of random processes [2]. This article attempts to lay the foundations for data science, using as a computational pillar the

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the advanced POD version-rooted in classical linear modal analysis, nonlinear normal modes-invariant manifolds theory, and experimental mechanics explorations [3]. Over the last ten years, we have conducted systematic lab and field experiments with complex structures to discover the quite important fact that the POD representation of properly selected acceleration datasets does not depend on the location the dataset is collected [4]. This occurs if the material is viscoelastic. Here, this empirical fact is taken as a ground reference situation and is refined into a principle to form the core of a data science framework. In this framework, the Advanced Proper Orthogonal Decomposition transform [3] is used as the primary tool to process raw datasets and derived ones. The competing, under certain conditions, technique of Experimental Modal Analysis (EMA) [5] is limited by the linearity factor and is unable to provide a reliable interpretation for dynamical processes in linear complex systems where nonlinearity due to wave chaotic scattering blurs the underlining linear dynamics with random uncertainty. It is well known that wave chaos can occur in linear structures whose domain continuity is interrupted by distributed holes [6,7]. Complex structures in applications are not smooth continua as they contain by design and damage precipitation and growth processes complicated interior and boundary heterogeneities. The ingredients are there for uncertainty in impulse-based nondestructive SH interrogations. Unconstrained by the nonlinearity or linearity of the dynamical process, the APOD packs these uncertain dynamics into physics-carrying modal patterns. This offers an unparalleled method for early-stage damage diagnosis in complex systems, such as multi-body structural systems.

OPTIMUM REDUCTION ANALYSIS: THE APOD TRANSFRORM

We step on the unparalleled computational order reduction of the APOD transform to introduce-and forge by experimenting with physical sensor-based datasets-elements of data science for damage diagnostics of complex structures with potential uncertainty. Being a 2-dimensional matrix $\mathbb{A}$: numerical values over the space-time domain discretized respectively by N and M points (assume N<M-no loss of generality), the typical wave-vibration dataset admits a natural unfolding in the intrinsic high-dimensional vector space, thus forming a point-data landscape (geometry) weaved around-and-along Intrinsic Maximum Correlations vector directions: the so-called proper orthogonal decomposition (POD) modes. The APOD Transform, a chain of linear computational operations on the dataset, detects and computes these POD modes. Specifically, the following expression [3, 8]:

$$\hat{\mathbb{A}} \equiv \sqrt{\frac{2}{M-1} \times \frac{2}{N-1}} \frac{\mathbb{A}}{\left\|\mathbb{A}\right\|} \xrightarrow[\text{decompose-project-verify-modal formation}]{\text{APOD}} \mathbb{A}_K \equiv \sum_{k=1}^{K} \hat{\mathbb{A}}_k, \qquad (1)$$
$$\mathbb{A}_K \xrightarrow[\text{modal reconstruction-convergence}]{\text{APOD}} \hat{\mathbb{A}} : \ 1 \leq K \leq K^* \leq N < M.$$

presents the K-th POD mode representation of the dataset. Matrix element $\hat{\mathbb{A}}_k$ represents the extracted POD modal pattern with mathematical structure:

$$\hat{\mathbb{A}}_k \equiv \sqrt{\hat{\lambda}_k} \hat{\mathbf{Q}}_k \hat{\mathbf{\Phi}}_k^{\mathbf{T}} : \text{ bi-orthogonality: } \hat{\mathbf{Q}}_k \bullet \hat{\mathbf{Q}}_m = \delta_{km} = \hat{\mathbf{\Phi}}_k \bullet \hat{\mathbf{\Phi}}_m. \qquad (2)$$

This factorized energy-time-space mathematical structure parallels the analytic modal analysis of linear systems wherein separation of variables is theorized and thus leading to the eigen-value problem reduction of linear mathematical models, opening the gate for analytic predictions on the basis of a model. APOD is an eigen-value problem-based computational process operating on the dataset level, thus not depending on the math model or the physical systems the dataset is originating from. Performing an optimum order reduction of the dataset, APOD opens the gate for approximation analytics and feature extraction from complex systems. Historically, POD was used to get physics insights of turbulence dynamical processes in fluid mechanics [9].

The goal of this article is to demonstrate the explore-discover-characterize attribute of the APOD transform of big datasets derived by a *union-augmentation* process on a set of datasets: here those stemming from the simultaneous collection of acceleration signals at three physical points, referred to as simultaneous collocated acceleration (SCAS) datasets. Among others, introduced is the concept of Spatial Differential Form (SDF) augmentation of a single CAS dataset. The POD mode of such an augmented dataset spreads across the first two spatial derivatives, thus naturally detecting changes in derivatives due to damage. In classical SHM studies [1,5], it is customary to seek structural damage evidence in curvature changes of normal bending modes. Here, by including attributes of differential geometry of the motion, as measured by physical sensors, we lay a data science framework for multiscale damage diagnostics in complex multi-body structures.

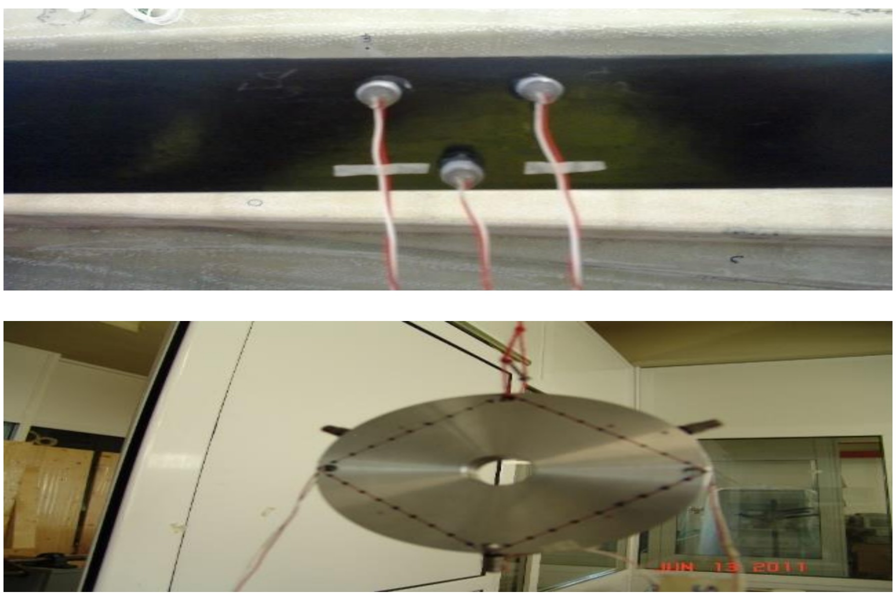

Figure 1. Solid mechanical structures instrumented with a triad of sensors to collect simultaneous datasets of acceleration response induced by a span wise impulsive force scanning. Top image: thin composite beam structure (regular structure). Bottom image: thick aluminum alloy rotor (machinery structure).

AUGMENTATION OF COLLOCATED ACCELERATION DATASETS

Figure 1 presents two paradigmatic structures with interior material and domain geometry irregularities, potentially blurring the wave-vibration free response with stochasticity-uncertainty at small scales, thus rendering the issue of damage detection a challenge: *How can we differentiate early-stage structural damage signs from uncertainty signs due to small scale wave chaos?* In a previous study [10], we observed that three SCAS datasets associated with the composite beam above share a number of POD modes. *This introduces the need to compute dataset intersections, the latter* *forming a novel way to extract motion invariant properties*. Nonlinear physics rolls in the possibility that a set of SCAS datasets for an arbitrary complex structure could be uncorrelated to a certain degree. Its domain continuity interrupted by a central hole surrounded by four planetary holes, the metallic rotor (Fig.1-bottom) is the test bed detecting wave chaos [7] in the generic complex structure, with a particular example, the composite structures considered here.

This we present by augmenting CAS datasets and reducing them to low-dimensional datasets. Specifically, given the CAS dataset at the location C, we compose its row wise union-augmentation with the simultaneously acquired CAS datasets at the locations L and R:

$$\mathbb{A}_{[\mathrm{LCR}]} \equiv \left[\mathbb{A}_{\mathrm{L}}, \mathbb{A}_{\mathrm{C}}, \mathbb{A}_{\mathrm{R}} \right]. \tag{3}$$

Referred to as the simultaneous 3-point observation (S3PO) scheme, this nontrivial dataset augmentation sets the math structure framework for a holistic physics-seeking exploration of the dynamics of a complex structure via simultaneous observations at multiple stations. Definitely, intersection of CAS datasets (point-data clouds in high-dimensions) should be sought in a physics-revealing sense and not in the classical set-theory sense, the point-wise one. This motivates steps with the APOD transform for a proper data science for structural dynamics.

Unparalleled, the APOD transform computes physics-dictated intersections among SCAS datasets as POD modal patterns (Fig. 2). Specifically, the generic k-th POD mode of the S3PO dataset admits the following energy-amplitude-shape math structure:

$$\mathbb{A}_{\mathrm{LCR\text{-}k}} \equiv \sqrt{\hat{\lambda}_{\mathrm{LRC\text{-}k}}}\, \hat{\mathbf{Q}}_{\mathrm{LCR\text{-}k}} \hat{\mathbf{\Phi}}^{\mathbf{T}}_{\mathrm{LCR\text{-}k}}. \tag{4}$$

Unit matrix $\hat{\mathbf{Q}}_{\mathrm{LCR\text{-}k}}$ denotes the POD modal amplitude waveform present in the totality of measurements packed to form the augmented dataset. Companion unit matrix $\hat{\mathbf{\Phi}}_{\mathrm{LRC\text{-}k}}$ represents the shape formed over the augmented space of points (3N). It is segmented dataset-wise to define the shape portions associated with the individual datasets:

$$\underbrace{\hat{\mathbf{\Phi}}^{\mathbf{T}}_{\mathrm{LCR\text{-}k}}}_{3\mathrm{N}} \rightarrow \left[\underbrace{\mathbf{\Phi}_{\mathrm{L\text{-}k}}}_{\mathrm{N}}, \underbrace{\mathbf{\Phi}_{\mathrm{C\text{-}k}}}_{\mathrm{N}}, \underbrace{\mathbf{\Phi}_{\mathrm{R\text{-}k}}}_{\mathrm{N}} \right]. \tag{5}$$

A strong intersection of the three SCAS datasets renders the segmented shapes analogous graphs, with periodicity the perfect analogy.

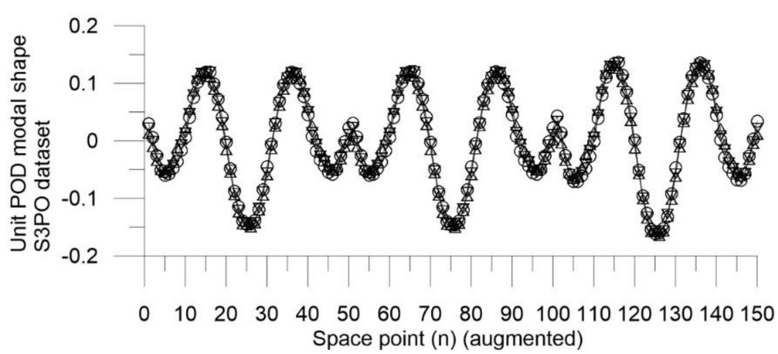

Figure 2 Shape of the dominant POD mode of three S3PO datasets of acceleration induced by three random span wise interrogation (various symbols). Composite beam structure excited at N=50 points.

Figures 2 and 3 reveal the quite remarkable result: for the composite beam the S3PO dominant POD modal shape (Fig.2) is completely analogous (in fact, nearly periodic);

whereas that for the metallic rotor (Fig.3) is not analogous (in fact, non-periodic). For the composite beam, the first eight POD modes of the S3PO datasets are analogous (near periodic); whereas all the POD modes of the S3PO datasets of the rotor structure are non-periodic.

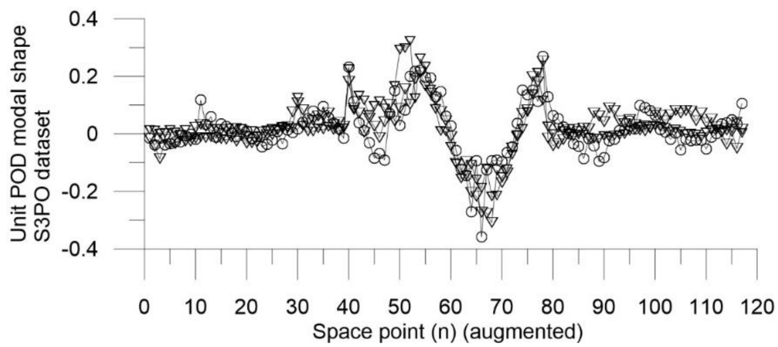

Figure 3. Shape of the dominant POD mode of three S3PO datasets of acceleration induced by three random span wise interrogation (various symbols). Aluminum alloy rotor excited at (N=39) points.

The high order modes (small energy and small scales) are non-periodic. Both the composite beam and the rotor structure have S3PO POD modes that are non-periodic. In both structures, the *non-periodic POD modes are sensitive to the profile of the SIE interrogation; whereas* the *periodic POD modes are robust.* Returning to Fig. 3, the dominant POD of the S3PO dataset for the metallic rotor is not only non-periodic but also non-smooth and, in addition to being sensitive to the SIE profile. The localization points to a propagating elastic wave which potentially undergoes chaotic scattering due to the nonlinearity scattering landscape formed by the geometry of the array of the five holes. This assertion is further cross-verified by the fact that the companion amplitude waveform is also localized in time.

Regarding the rotor structure with holes, where theory predicts wave scattering chaos [6,7], the APOD transform revealed that a S3PO dataset is resolved into POD modes: all non-periodic (non-analogous) and non-smooth. On the basis of this result, the high-order non-periodic and non-smooth POD modes for the composite beam S3PO dataset point to chaotic wave scattering. Precipitating uncertainty, it obscures the diagnosis of structural damage at small scale.

DIFFERENTIAL GEOMETRIC IDENTIFICATION & DIAGNOSTICS

In this section we augment a dataset to include attributes of differential geometry aimed at innovative advanced diagnostics for data-driven structural health monitoring. Figure 4 depicts two composite beams synthesized by a pressure-temperature-controlled molding process for manufacturing structures [11]. The material is high-tech continuous fiber soaked in a matrix made of polymeric material. During the manufacturing process, several factors can lower the quality of the synthesized structure, for example, trapping air bubbles and heterogeneity in the interfaces of the fibre-polymer layers the structure is synthetized from. A local triad of sensors is used to form a database of collocated acceleration datasets. The scanning force covers the structure spam wise.

Given a reference dataset $\mathbb{A}$ of collocated acceleration signals, the Spatial Differential Form (SDF) augmented dataset is defined to be the row wise union of three blocks of data:

$$\mathbb{A}_{[\mathrm{SDF}]} \equiv \left[\mathscr{D}_0[\mathbb{A}], \mathscr{D}_1[\mathbb{A}], \mathscr{D}_2[\mathbb{A}] \right]. \tag{6}$$

The matrices, blocks of data, are defined as follows:

$$\mathscr{D}_0\left[\mathbb{A}\right] \equiv \mathbb{A} \equiv \left[\mathrm{a}_1, \mathrm{a}_2, \ldots, \mathrm{a}_n, \mathrm{a}_{n+1}, \ldots, \mathrm{a}_{N-1}, \mathrm{a}_N \right],\ \mathrm{D}_0 = [1,2,\ldots,\mathrm{N}],$$

$$\mathscr{D}_1\left[\mathbb{A}\right] \equiv \left[\mathrm{a}_2 - \mathrm{a}_1, \ldots, \mathrm{a}_{n+1} - \mathrm{a}_n, \ldots, \mathrm{a}_N - \mathrm{a}_{N-1} \right],\ \mathrm{D}_1 = [1,2,\ldots,\mathrm{N}-1],$$

$$\mathscr{D}_2\left[\mathbb{A}\right] \equiv \left[\mathrm{a}_3 - 2\mathrm{a}_2 + \mathrm{a}_1, \ldots, \mathrm{a}_{n+2} - 2\mathrm{a}_{n+1} + \mathrm{a}_n, \mathrm{a}_N - 2\mathrm{a}_{N-1} + \mathrm{a}_{N-2} \right], \mathrm{D}_2 = [1,2,\ldots,\mathrm{N}-2].$$

Column vector a_n represents the M-points time series measurement the fixed sensor detects for the induced motion an impulsive force excites at location n .

Figure 4. Suspended continuous carbon fiber composite beams of length 1500 mm instrumented with a local trial of piezoelectric accelerometer. Top picture: uniform rectangular cross-sections. Bottom picture: T-shaped uniform cross-section. A statically scanning impulsive force scans (manual) the structures.

Dataset SDF augmentation is quite natural mathematically and physically: it is the mathematical form endowing a dataset with its differential geometry. It has been observed that structural damage affects the curvature of bending modal vibration shapes [1,5]. Let vector $\hat{\mathbf{\Phi}}_{\mathrm{SDF\text{-}k}}$ denote the k-th POD modal shape of a specific SDF-dataset associated with the structures shown above. The POD modal shape is restricted on the three blocks of the augmented domain of definition as follows:

$$\mathscr{P}_0\left[\hat{\mathbf{\Phi}}_{\mathrm{SDF\text{-}k}}\right] \to \mathbf{\Phi}_k^{[0]},\quad \mathscr{P}_1\left[\hat{\mathbf{\Phi}}_{\mathrm{SDF\text{-}k}}\right] \to \mathbf{\Phi}_k^{[1]},\quad \mathscr{P}_2\left[\hat{\mathbf{\Phi}}_{\mathrm{SDF\text{-}k}}\right] \to \mathbf{\Phi}_k^{[1]}, \tag{7}$$

to obtain the differential order-0,1,2 modal shapes, all defined as distributions on the base space of excitation points: [1, 2, …, N]. These are respectively representations of the POD modal shape and its first and second derivatives.

Figure 5 presents the shapes of the dominant POD mode (differential order-0 restriction) for the beam with a rectangular cross-section (Fig.5-top) and the beam with a T-shaped one (Fig.5-bottom). Both modal shapes admit smooth curve fittings by convergent polynomials (solid curve). It is remarkable (known for metallic beams) that these POD modal shapes are related to natural bending modes. This is where the APOD transform meets the classical experimental modal analysis (EMA) technique of linear dynamics, thus pinpointing collocated acceleration datasets as basic for data science.

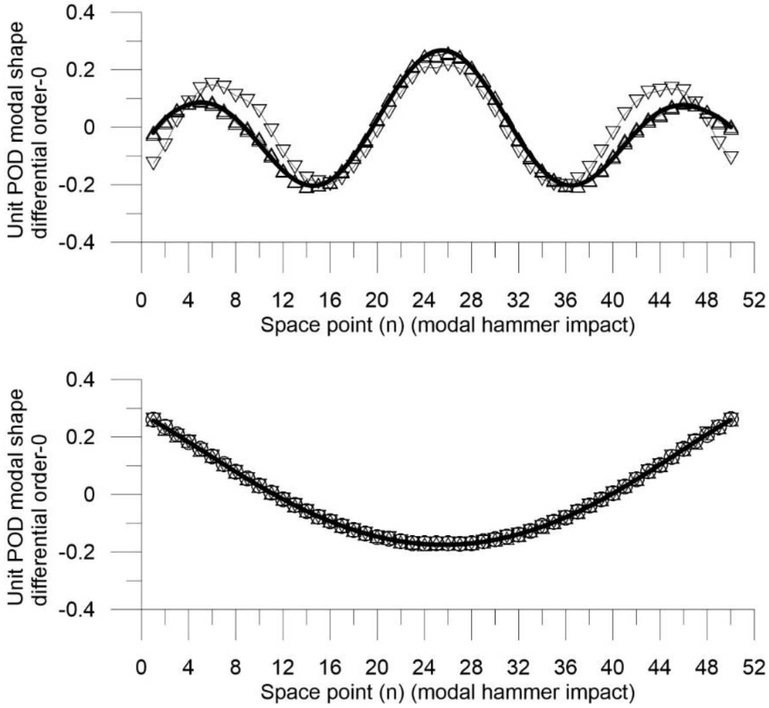

Figure 5. Dominant POD mode of an SDF-dataset (three samples): distribution of the differential order-0 restriction**.** Top: composite beam of rectangular cross-section. Bottom: composite beam of T-shaped cross-section.

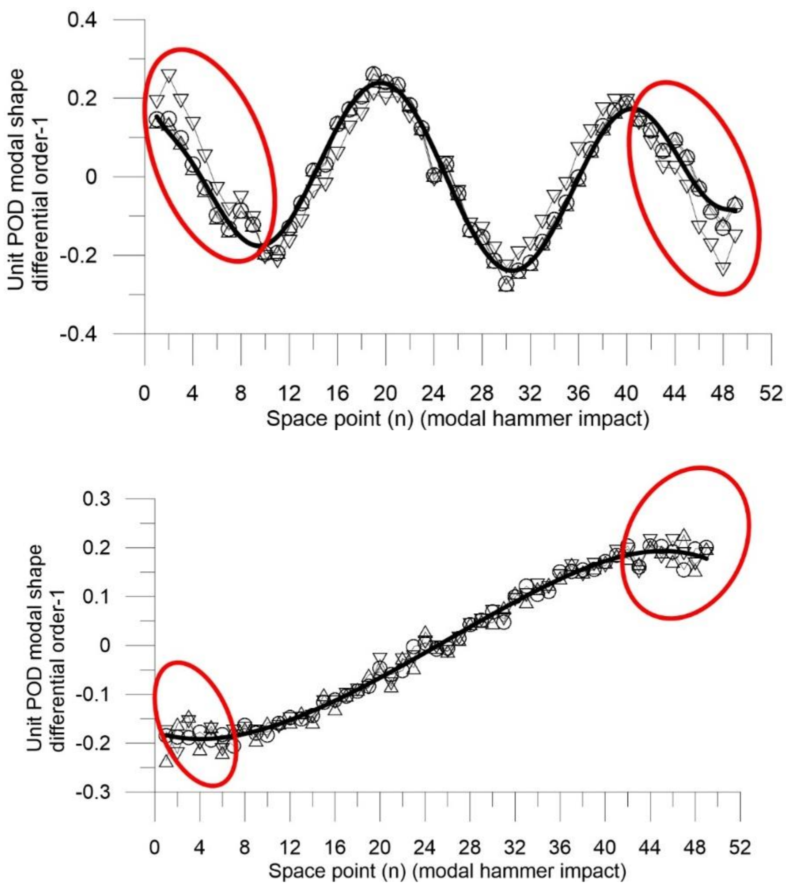

Figure 6. Dominant POD mode of an SDF-dataset (three samples): distribution of the differential order-1 restriction. Top: composite beam of rectangular cross-section. Bottom: composite beam of T-shaped cross-section.

Operating on CAS datasets, APOD reveals how the Reciprocity Principle [12] unfolds in the weak nonlinear regime in the presence of geometric nonlinearity intrinsically present in wave chaotic scattering blurring the dominant linear dynamics. *Were the tested structures ideally linear-with no mechanism to cause wave chaos-the data points should collapse into a single smooth curve*. The three closely packed curves pinpoint the physics behind the dominant POD nodal shape.

Figure 6-top presents the shape of the first derivative (differential order-1 restriction) of the dominant POD mode (differential order-0 restriction) for the beam with rectangular ross-section. The best curve fitting is identified with the first derivative of the best curve fitting in Fig. 4-top Figure 6-bottom presents the shape of the first derivative (differential order-1 restriction) of the dominant POD mode (differential order-0 restriction) for the beam with T-shaped cross-section (bottom). Noticed are the first signs of no smoothness of the shape of the dominant POD mode: shape fluctuation of the two boundary regions (red circles). This is the well-known boundary effect.

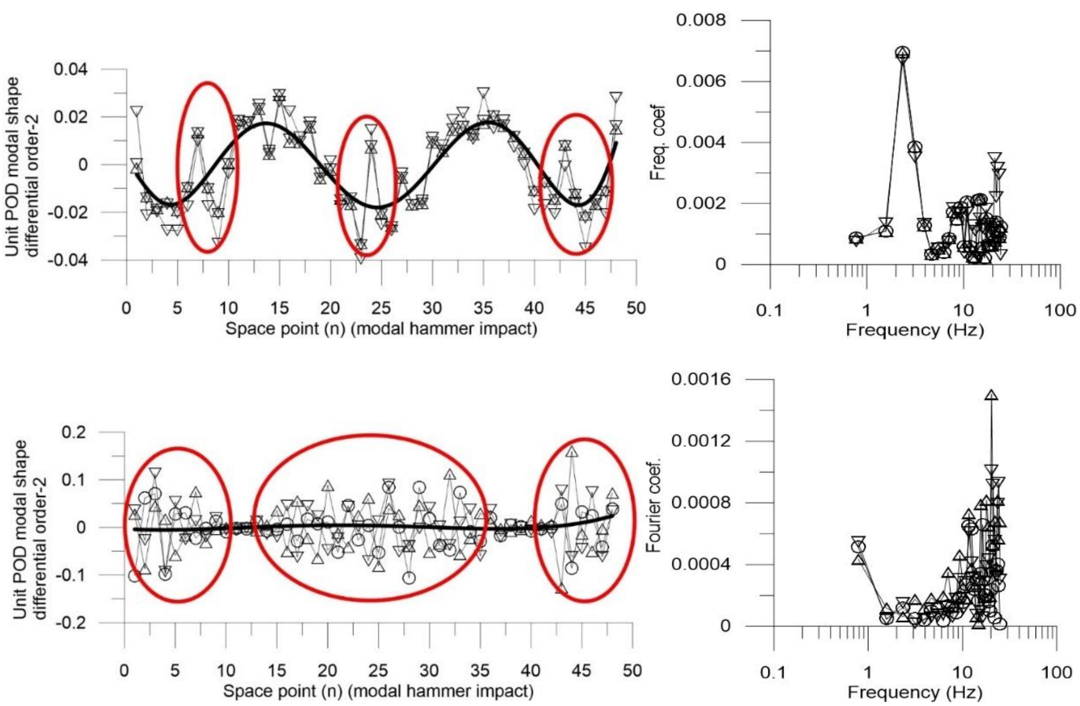

Figure 7. Dominant POD mode of an SDF-dataset (three samples): Differential Order-2 restriction (left), FFT (right). Top: rectangular cross-section beam. Bottom: T-shaped cross-section beam.

Figure 7 presents the second derivative (differential order-2 restriction) of the dominant POD mode (differential order-0 restriction) for the rectangular cross-section beam(top) and the T-shaped cross-section beam (bottom). Both shapes are best fitted by well-behaved polynomials which are the second derivatives of the fitting polynomials shown in Fig. 4. In both cases, we notice regions at the boundaries and in the interior with intense shape fluctuations (marked red circles). Figure 7-right reveals that the three samples collapse into a single curve in the wave number domain (FFT). Noticed are the slow spatial frequency (smooth polynomial) and higher order spatial frequencies. The fast variation is sensitive to damage. APOD of SDF datasets leads to a revealing multiscale analysis: slow and fast spatial variations. The striking fact of the APOD of the SDF3 dataset is the one-step computation of the norms of the POD modal shape and its first two derivatives. Specifically, the norms of the dominant POD mode and its first two derivatives are distributed respectively as [0.990024, 0.138180, 0.027148] for the T-shaped beam and as [0.9429850, 0.307582, 0.124302] for the rectangular beam. The ratio of the norm of the first derivative to that of the POD mode characterizes the *relative shearing flexibility*; whereas the ratio of the norm of the second derivative to that of the POD mode characterizes the *relative bending flexibility*. Consequently, an identification-and-damage diagnosis spectrum of relative flexibilities is derived over the wave number of the POD modes.

We have the following remarkable result: Direct computations reveal that the APOD transform of the individual datasets, $\mathscr{D}_0[\mathbb{A}], \mathscr{D}_1[\mathbb{A}], \mathscr{D}_2[\mathbb{A}]$, composing the differential augmentation, led to the remarkable result:

$$\mathbf{\Phi}_k^{[0]} \approx \mathscr{D}_0\left[\hat{\mathbf{\Phi}}_k\right],\ \mathbf{\Phi}_k^{[1]} \approx \mathscr{D}_1\left[\hat{\mathbf{\Phi}}_k\right],\ \mathbf{\Phi}_k^{[2]} \approx \mathscr{D}_2\left[\hat{\mathbf{\Phi}}_k\right], \tag{8}$$

where $\hat{\mathbf{\Phi}}_k$ is the k-th POD modal shape of the reference dataset $\mathbb{A}$. This verifies the invariance-preservation of the geometry of motion under the APOD transform of the differential augmentation of a motion dataset. The APOD analysis of differential augmentation brings in differential geometry, namely tangencies to formulate broad damage diagnostics. The data science aspect is the deep APOD resolution of the SDF-dataset.

DISCUSSION AND CONCLUSIONS

In facing the issues of sustainable mobility and autonomy of the complex transportation system, a multi-body assembly of elementary structures with coupled machinery, the great challenge is the extraction of reliable identification along with damage diagnostics information from a multichannel dataset stemming from distributed sensors. Here, we focus on acceleration signals detected by metrology standards-certified piezoelectric accelerometers. Classical Experimental Modal Analysis can provide some reliable answers, provided that the system is restricted to responding in the very weak nonlinear regime. The domain complexity, in the form of distributed material heterogeneities, joints, and damage in the form of cracks and corrosion, will blur the datasets with uncertainty even if the structure is behaving nearly linear. Here we presented the uncertain dynamics of an aluminum alloy rotor percolated with five holes. One source of detectable uncertainty is that routed to the phenomenon of wave chaotic scattering. A database of collocated acceleration signals has the potential to overcome the above challenge, since under certain conditions these datasets provide the POD representation of linear normal modes of vibration. This case is taken as a reference, or even an axiom, to apply the APOD transform to tensors to extract useful information from a database of wave-vibration datasets in the unrestricted presence of uncertainty. The introduced S3PO augmentation of datasets, here we experience its capability to detect invariants of the wave-vibration motion, can be generalized to detect-compute coherence or no-coherence among datasets collected at separate strategic observation locations. The no-coherence we detected was due to the phenomenon of wave chaotic scattering which seems to be present in the high order POD modes of carbon fibre composite beams. The SDF augmentation of a dataset, here it leads us to compute pure data-driven representation of the second derivative of the POD modal shapes of composite beams as the footprint of the state of structural health status as sealed by the manufacturing process, can be generalized to any complex structure to seek and compute damage indication in local critical areas of complex multi-body structural systems. The success of the above augmentations has formed the impetus for the systematic development of data science for mechanics by relying on the unparalleled order reduction provided by the advanced proper orthogonal decomposition for tensor datasets. For example, important datasets of tensor structure stem from the triaxial accelerometer device and the inertia measurement unit device used in marine and aerospace applications. In our present research endeavor, we are exploring other continuum mechanics and computational mechanics-compatible augmentations of basic datasets.

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