Compressed Dynamic Mode Decomposition for Full-Field Modal Identification from Video Measurements

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ABSTRACT

Traditional sensor-based modal identification methods can be intrusive, costly, and limited by sensor numbers and placement. This research presents an end-to-end vision-based automated full-field modal identification framework that overcomes these limitations by combining a Phase-Based Optical Flow (PBOF) method and a Hankel Compressed Dynamic Mode Decomposition (HCDMD) approach. The proposed HCDMD approach automates the full-field modal identification process with high efficiency by employing a compressed sensing strategy to accelerate computation and a Local Standard Deviation (LSD) index to separate structural and noise mode shapes. A laboratory experiment was utilized to validate the effectiveness and efficiency of the proposed framework in measuring full-field displacement and extracting modal parameters. The results demonstrate the potential of this fully automated end-to-end solution for full-field modal identification from video measurements.

INTRODUCTION

Structural Health Monitoring (SHM) has been a critical aspect of ensuring the safety, reliability, and longevity of civil infrastructure. Among the various SHM techniques, modal identification, the process of determining the dynamic characteristics of a structure, such as natural frequencies, mode shapes, and damping ratios, plays a pivotal role in assessing the structural integrity and detecting anomalies that may indicate potential failures. Traditional modal identification methods often rely on sensor-based data acquisition, which, while effective, can be intrusive, expensive, and limited by the number and placement of sensors. With advances in computer vision, video-based approaches have emerged as a promising solution for structural inspection and monitoring [1]. Especially, vision-based motion estimation techniques offer a non-contact, full-field, and cost-effective means of measuring structural vibrations. Among these, the Phase-Based Optical Flow (PBOF) algorithm has shown superior robustness to variations in contrast, scale, orientation, and motion speed compared to traditional intensity-based methods [2-3].

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Modal identification using full-field displacement is a critical post processing step, as the identified modal properties can directly serve as indices to assess the health condition of civil infrastructure. Chen et al. [2] proposed using PBOF combined with motion magnification techniques to visualize mode shapes of a cantilever beam. The method can reveal subtle movements but requires manual selection of resonant frequency peaks and lacks quantification. Yang et al. [4] further employed blind source separation to address these issues, but the technique still necessitates human intervention for mode quantification and selection. Bhowmick et al. [5] employed the Hankel Dynamic Mode Decomposition (HDMD) algorithm for linear dynamic system identification and verified that the HDMD can identify the latent dynamic modes similar to the Eigensystem Realization Algorithm (ERA). However, constructing and decomposing the Hankel-type matrix is time consuming, especially considering the extensive volume of measurement points in full-field video measurements. Moreover, manual separation of true structural modes from noise modes in the identification results is cumbersome.

In this research, a vision-based automated full-field modal identification framework is presented to address the above concerns. First, the full-field structural displacement is measured using the PBOF method. Subsequently, automated modal identification is conducted on the measured displacement field using the proposed Hankel Compressed Dynamic Mode Decomposition (HCDMD) approach. A laboratory experiment was employed to validate the performance of the proposed approach in full-field modal identification.

In the following, Section 2 gives a detailed description of the proposed vision-based automated full-field modal identification framework. Section 3 experimentally demonstrates the efficacy of the proposed methods in extracting modal information from the video measurement. Section 4 presents the conclusion and future works.

METHODOLOGY

Figure 1 shows the overview of the proposed vision-based automated full-field modal identification framework. The method consists of two main steps, namely a vision-based full-field displacement measurement step followed by an automated dynamic mode extraction step. In this study, the Phase-Based Optical Flow (PBOF) algorithm is employed for displacement measurement. For brevity, the detailed implementation of the PBOF algorithm is omitted here; interested readers are referred to [2,3,6] for further information.

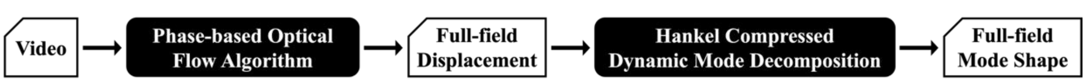

Figure 1. Proposed vision-based automated full-field modal identification framework.

The dynamic mode decomposition (DMD) algorithm was first introduced to extract spatiotemporal modes from fluid dynamic measurements. DMD is a purely data-driven method and has been demonstrated to approximate the Koopman operator for non-linear dynamic analysis [7]. In this study, the Hankel Compressed Dynamic Mode Decomposition (HCDMD) approach is proposed to achieve full-field dynamic modes extraction from video measurements with high accuracy, efficiency, and full automation. The overview of the method is presented in Figure 2 and details of each component are given in the following subsections.

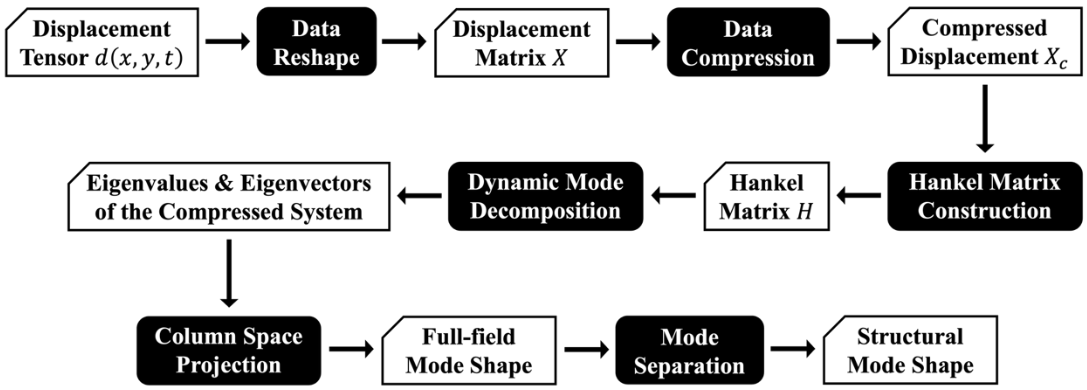

Figure 2. Overview of the proposed Hankel compressed dynamic mode decomposition method.

DISPLACEMENT DATA PREPROCESSING

The displacement tensor $d(x, y, t)$ is first reshaped into a two-dimensional matrix X_0 with the columns representing the time step and rows indicating the pixel locations:

$$X_0 = \begin{bmatrix} | & | & | & | \\ x(1) & x(2) & \cdots & x(t) \\ | & | & | & | \end{bmatrix} \tag{1}$$

where $x(t)$ is a n-dimensional displacement vector measured at time t; n denotes the number of pixels.

The standard deviations of each row in the displacement matrix are then computed, and those rows with a standard deviation less than 0.05 pixels are discarded. The size of the displacement matrix X is thus reduced to $m \times t$, where m represents the number of measurement points ($m \ll n$). This step removes displacement signals originating from stationary background features or low SNR regions determined by the weight matrix W_θ, thereby reducing computational costs.

The obtained displacement matrix X is still of high dimension but sparse in a transform basis Ψ, which can be expressed by

$$X = \Psi \mathbf{s} \tag{2}$$

where $\mathbf{s}$ is a sparse vector with K nonzero elements.

Equation (2) indicates that the full-field structural displacement exhibits a high-degree of compressibility. Additionally, Brunton et al. [7] has demonstrated that the eigenvalues of DMD on compressed data are equivalent to the full-state DMD eigenvalues if the measurement matrix is chosen properly to be incoherent with respect to the transform basis and contains sufficient measurements. Candes and Tao [8] mathematically proved that a Gaussian random measurement matrix can be a suitable choice, as it can satisfy the restricted isometric property with high probability for a generic transform basis. Therefore, the reduced displacement matrix can be further compressed using a Gaussian random measurement matrix C, as represented by

$$X_C = CX \tag{3}$$

where X_C is the compressed displacement matrix with a dimension of $p \times t$, and $K\log(m/K) \leq p \leq m$ is set to ensure that X_C contains sufficient linearly independent displacement signals to capture the dynamic information of the displacement matrix.

The data compression ratio can be computed as

$$\text{compression ratio} = \frac{p}{m} \times 100\% \tag{4}$$

AUTOMATED DYNAMIC MODE EXTRACTION AND SEPARATION

In this step, the obtained compressed displacement matrix is used to construct the Hankel matrix for dynamic mode extraction. The Hankel matrix block H_m corresponding to the m^{th} measurement point can be represented as

$$H_m = \begin{bmatrix} x_m(1) & x_m(2) & \cdots & x_m(k) \\ x_m(2) & x_m(3) & \cdots & x_m(k+1) \\ \vdots & \vdots & \ddots & \vdots \\ x_m(q) & x_m(q+1) & \cdots & x_m(k+q-1) \end{bmatrix} \tag{5}$$

where q and k denote the number of rows and columns in the Hankel matrix block, respectively. Assembling the matrix block of each measurement point yields the Hankel matrix H:

$$H = [H_1 \quad H_2 \quad \cdots \quad H_p]^T \tag{6}$$

The modal information can then be uncovered by solving the process matrix A_H:

$$A_H = H'H^{\dagger} \tag{7}$$

where A_H is a $pk \times pk$ process matrix, also known as the DMD operator; H' is the displacement matrix H shifted one time step later; $H^{\dagger}$ is Moore-Penrose pseudoinverse of the displacement matrix H, which can be derived from the singular value decomposition (SVD) of H:

$$\begin{cases} H \approx U_r \Sigma_r V_r^T \\ H^{\dagger} \approx V_r \Sigma_r^{-1} U_r^T \end{cases} \tag{8}$$

where U_r and V_r are the truncated left and right unitary matrices of sizes $q \times r$ and $pk \times r$, respectively; r is the truncation rank of the singular values Σ.

By substituting Equation (8) into (7) and performing eigenvalue decomposition on the DMD operator A_H, the eigenvalues Λ and eigenvectors W of the system can be obtained by

$$A_H W = W\Lambda \tag{9}$$

Since the eigenvalues obtained using the compressed displacement data are equivalent to the full-state DMD eigenvalues, the structural modal frequency and damping ratio can be directly computed via

$$f_i = \mathrm{Im}\left(\frac{\ln\lambda_i}{2\pi\Delta t}\right) \tag{10}$$

$$\varsigma_i = \mathrm{Re}\left(\frac{\ln\lambda_i}{2\pi\Delta t}\right)/f_i \tag{11}$$

where f_i and ς_i are the i^{th} structural modal frequency and damping ratio, respectively; λ_i is the i^{th} eigenvalue; $1/\Delta t$ is the sampling rate.

Note that the eigenvectors W are derived from the compressed system. To obtain the full state structural mode shapes Φ, eigenvectors need to be projected to the column space of the original displacement matrix, which can be represented by

$$\Phi = X'V\Sigma^{-1}W \tag{12}$$

where X' is the displacement matrix X shifted one time step later.

The modal properties obtained using Equations (10) to (12) include not only structural modes of interest but also noise modes induced by measurement error or potential background movements. Figure 3 illustrates an example of a structural mode shape and a noise mode shape of a six-story shear building model, showing that noise modes typically exhibit higher randomness compared to structural modes. Based on this, a Local Standard Deviation (LSD) index is proposed herein to quantify the local randomness of the obtained 2-D mode shapes, as depicted in Figure 4. The dotted box in Figure 4 outlines the LSD index computation workflow for each pixel (x, y) with a valid mode shape value in a single mode. The neighborhood size is defined to be 9×9 pixels, which is equivalent to the complex filter size. The average LSD of each mode is then computed and sorted in ascending order. A significant difference in the average LSD values can be observed between structural modes and noise modes. Modes with lower averaged LSD values correspond to structural modes, while higher values indicate noise modes.

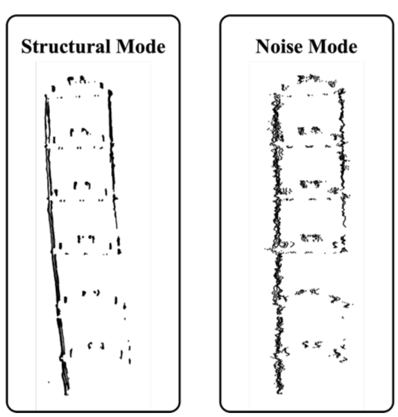

Figure 3. Example of structural mode shape (left) and noise mode shape (right).

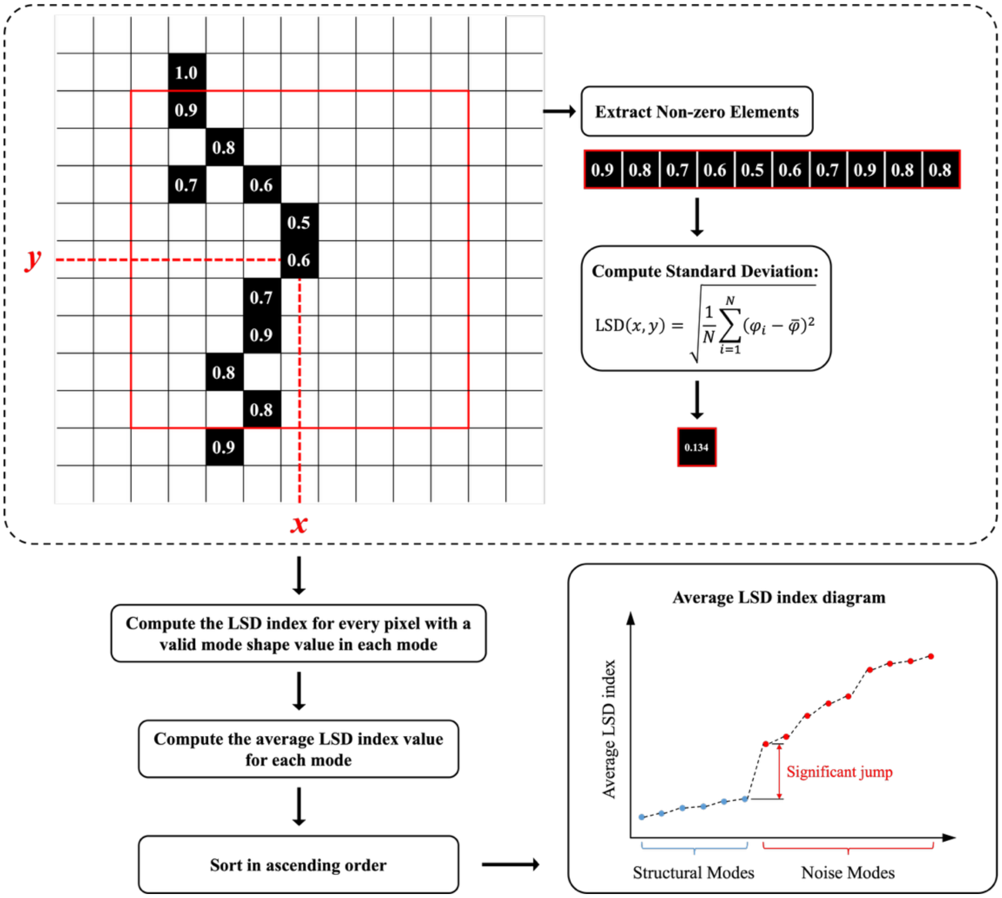

Figure 4. Proposed local standard deviation index for mode separation.

LABORATORY TEST VALIDATION

A 28-second video (1570×430p, 59.94fps) of a six-story shear building model with free vibration is used herein to verify the proposed automated full-field modal identification. This steel building model has been used for validation in previous studies on vision-based measurement [9]. The weight of each story is approximately 2.8kg and the first six modal frequencies measured by the accelerometers (in Hz) are 1.657, 5.038, 8.138, 10.833, 12.930 and 14.339.

First, the full-field displacement of the structure was measured using the PBOF algorithm. Figure 5 (left) shows the estimated displacement field of the right column of the structure, revealing a clear free damped vibration pattern. Subsequently, the algorithm applied a pixel mask to select high SNR regions and remove stationary background objects based on the estimated displacement field, as described in section 2.2.1. This step reduced the number of valid measurement points from 677101 pixels to 10341 pixels, as illustrated in Figure 5 (right).

By further compressing it with a compression ratio of 5%, the displacement matrix was reduced from a size of 10341×1700 to 517×1700. The proposed HCDMD algorithm was then utilized to identify the modal properties of the compressed system and project the eigenvectors back to the full-state measurement. Figure 6 presents the average LSD index diagram of the first 20 modes and a significant jump can be observed between fourth and fifth mode, indicating that the first 4 modes are structural modes. However, the fifth and sixth structural modes fail to be identified because they are not noticeably excited in the free vibration video obtained.

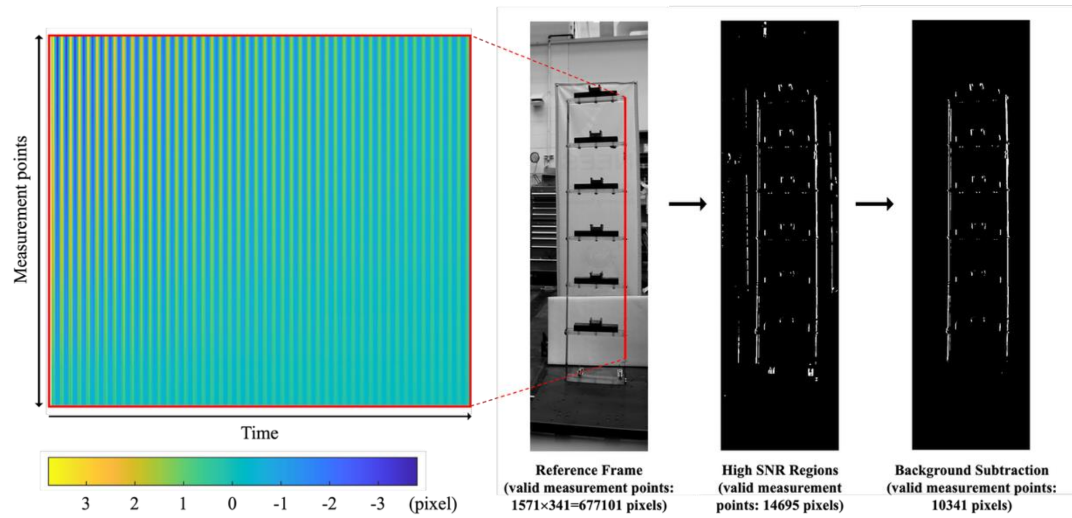

Figure 5. Full-field displacement measurement result and data preprocessing.

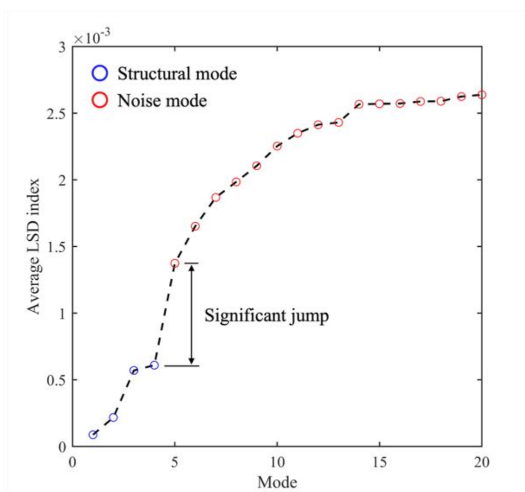

Figure 6. Average LSD index diagram.

Table 1 demonstrates the close match between the modal frequencies and damping ratios estimated by the proposed HCDMD method and the reference obtained using ERA applied on the accelerometer measurements. The result shows that the modal frequencies measured by the accelerometer are around 1% smaller than the HCDMD results. This discrepancy may partly arise because the accelerometer-based and vision-based measurements are conducted independently, and the self-weight of the accelerometers can influence the measurement. The four identified full-field structural modes are visualized in Figure 7, including a comparison of the reference mode shapes measured by the accelerometers. The corresponding Modal Assurance Criterion (MAC) values all exceed 99%, showing high accuracy. Note that the above process is fully automated and does not require any human intervention, highlighting the great potential of the proposed method in infrastructure monitoring automation. Additionally, the proposed method achieves a computation time of 12.8 seconds on a PC equipped with a 3.19 GHz Intel i7 processor and 16 GB of RAM, compared to 1467 seconds required by the conventional HDMD approach on the same device. These results highlight the substantial reduction in computational cost enabled by the proposed compression strategy, further demonstrating the accuracy, robustness, and efficiency of the HCDMD method.

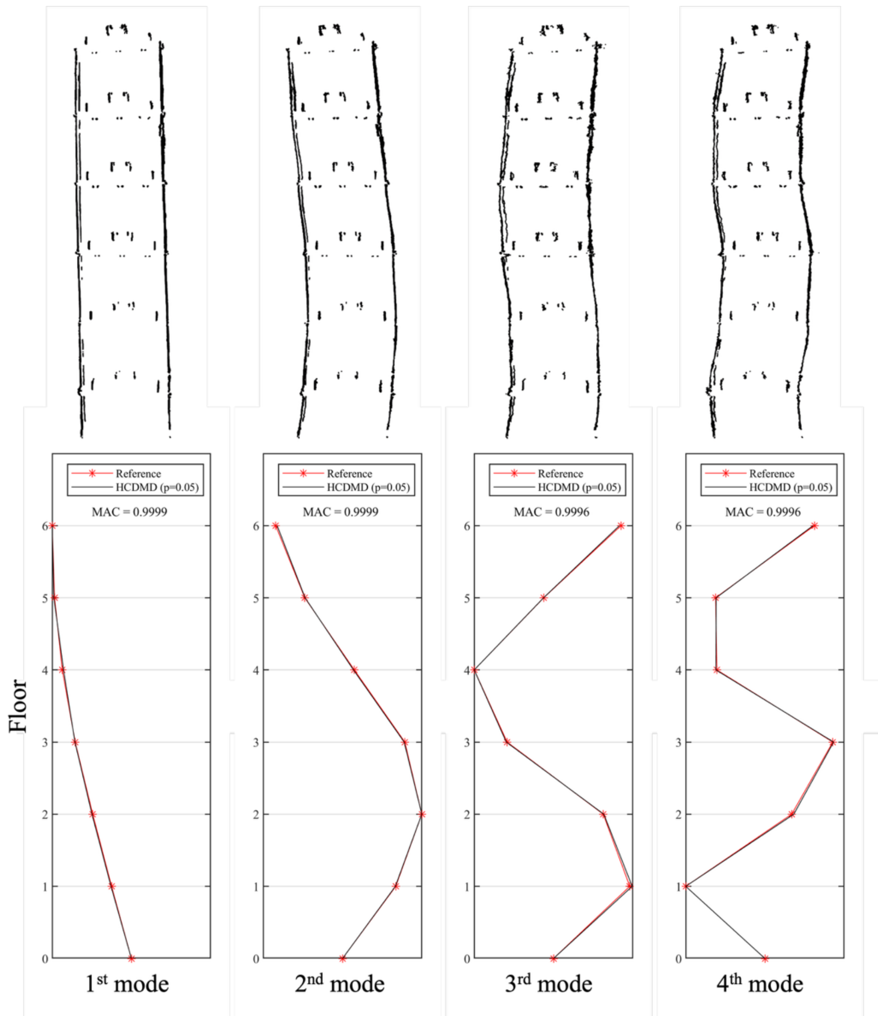

Figure 7. Full-field structural modes identified by the proposed HCDMD method.

TABLE I. MODAL IDENTIFICATION RESULTS BY THE PROPOSED APPROACH

Mode	Modal frequency (Hz)		Modal damping ratio (%)	
	Reference	HCDMD	Reference	HCDMD
1	1.657	1.667	0.47	0.42
2	5.038	5.087	0.39	0.40
3	8.138	8.239	0.33	0.42
4	10.833	10.908	0.32	0.35

CONCLUSION

This paper presents a vision-based automated full-field modal identification framework, addressing the critical need for a non-contact, full-field, and automated solution for structural health monitoring (SHM). The proposed Hankel compressed dynamic mode decomposition (HCDMD) approach has streamlined the full-field modal identification process by employing a compressed sensing strategy, significantly reducing computation time while maintaining high accuracy. The introduction of the Local Standard Deviation (LSD) index has automated the separation of structural and noise mode shapes, making the framework suitable for infrastructure monitoring automation.

The laboratory experiment conducted in this study has validated the effectiveness and efficiency of the proposed framework. The modal parameters determined by the HCDMD algorithm align well with those computed using the Eigensystem Realization Algorithm (ERA) applied to accelerometer measurements. The modal identification speed was accelerated up to hundreds of times, showing the computational efficiency of the proposed methods.

In conclusion, the high accuracy, efficiency, and automation capabilities of the proposed framework demonstrate great potential for the future of civil infrastructure monitoring, enabling more effective assessment of structural integrity. Future research can focus on exploring the framework's applicability to videos captured by non-stationary cameras, such as unmanned aerial systems.

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