

Leveraging Fractal Geometries to Improve Structural Health Monitoring Systems

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ABSTRACT

Fractal geometries are characterized by interesting properties that have not yet been exploited for structural health monitoring (SHM). Among such properties, scale invariance and self-similarity are worth mentioning. We investigated whether using fractal geometry to develop sensors or self-sensing structures can enhance SHM performance. The latter solution brings about the possibility of integrated damage diagnosis by eliminating the need for external sensors.

The theory of fractals was used, along with numerical simulations and experimental tests, to evaluate the feasibility of designing sensors and structures with fractal properties for enhanced SHM performance. In particular, a unique solution to develop a sensor or structural component based on the Sierpinski triangle was studied. Due to its hierarchical structure, the considered fractal geometry inherently possesses frequency domain characteristics that make it highly sensitive to local variations across scales.

Preliminary results brought evidence that fractals can be exploited to design high-performance SHM solutions, either as standalone fractal sensors or as self-sensing fractal structures. In addition, the efficient, lightweight, and multiscale nature of these systems makes them suitable for applications in several fields, including aerospace, mechanical, and biomedical engineering.

This work contributes to the advancement of SHM technologies by introducing a new paradigm in sensor design and damage detection. The combination of fractals with engineering applications shows the feasibility of advancing engineering by integrating complex physics principles and has potential implications for a wide range of industries.

INTRODUCTION

Fractal geometries are shapes that exhibit geometric self-similarity across different scales and are characterized by scale invariance properties. Since Mandelbrot introduced the formal theory of fractals in 1983 [1], the presence of such geometries in nature has

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been extensively documented. For example, fractal patterns can be found in trees, blood vessels, and geological formations [2, 3]. Inspired by these findings, fractal designs have also gained attention in engineering due to their hierarchical structure and scale-invariant properties [4, 5].

Despite increasing interest in such complex geometries, little work is available in the field of structural health monitoring (SHM). In fact, to date, the only few contributions that utilize fractal theory for damage diagnosis are based on the use of fractal analysis and fractal dimension, while fractal sensors have not been proposed so far [6–10]. Concurrently, few contributions have shown that structures designed with fractal architecture provide tunable, interesting mechanical properties, and energy absorption capabilities [11–13], although such structures have not yet been adopted or tested in damage-tolerant applications.

Building on the few preliminary studies in the literature, this work aims to study the properties of fractals and to identify if they may be used to advance SHM capabilities, offering compact, lightweight, and high-sensitivity solutions. In this study, we consider a representative fractal geometry, namely the Sierpinski triangle. This choice was made because the architecture has recently been found in nature [14] and has already been used in other fields with promising results [15, 16]. The structure was found to exhibit multiscale spectral properties and multiple frequency bandgaps. Moreover, a deeper investigation allowed us to connect the bandgaps to damage properties, thus bringing evidence of the potentialities of the fractal architecture when used to create sensors or integrated self-sensing structural components. Our hypothesis is that changes induced by damage will manifest distinctly in the spectrum across different scales, making the structure inherently capable of self-diagnosing damage. Furthermore, the Sierpinski triangle’s high porosity ensures lightweight construction without compromising stiffness, ideal for applications in aerospace, civil, mechanical, and biomedical engineering.

SIERPINSKI TRIANGLE

The Sierpinski triangle, also known as the Sierpinski gasket, is a fractal architecture characterized by geometric self-similarity. That is, systematically modifying the generational order of the system results in an identical geometric pattern. In the case of a perfectly scale-invariant Sierpinski, there exist infinitely many generational orders. However, for practical applications, due to the exponential growth of the system, it is necessary to naturally limit its size. In such a finite Sierpinski structure, each generational scale, or generation, is defined as an iterative step in the scaling process, as depicted in Fig. 1. Starting from the fundamental unit, generation 1, having side length L_0 , which is twice the smallest beam length, three replicas are combined to form generation 2, with side length $2L_0$. This procedure is repeated iteratively until the desired generation is achieved.

If we consider $B(g) = 3^{g+1}$ beams in a generation g Sierpinski, then we arrive at the characteristic length scale factor of the generation, Λ_g , by normalizing this quantity by $B(1)$ and relating the two quantities by a *power law*, as shown in Eq. 1:

$$\frac{B(g)}{B(1)} = 3^{g-1} = \Lambda_g^d \quad (1)$$

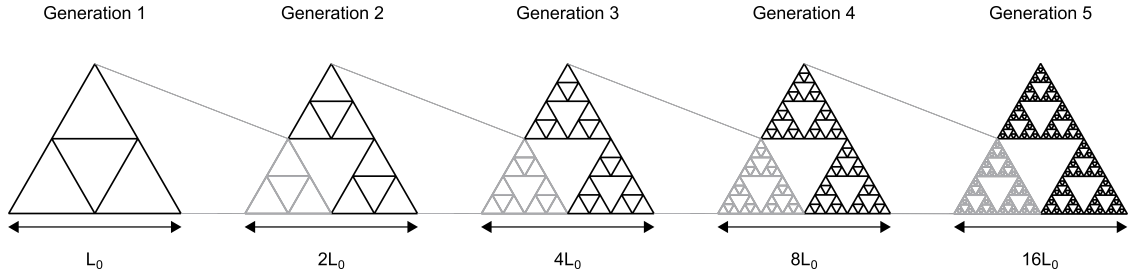


Figure 1. Construction of a 5-generation Sierpinski triangle by addition. We begin with generation 1, which is copied three times in order to scale the system size up by a factor of 2. This process is repeated iteratively by generation until the desired generation is reached.

where the power d is the fractal, or Hausdorff, dimension of the self-similar geometry. In this case, we have $d = \log 3 / \log 2 \approx 1.585$. The characteristic length (edge length) of generation g is then given by $L_g = L_0 \Lambda_g$. We note that generation 1 is simply a finite triangular lattice, making it the dimensional exception with $d = 2$.

We conducted a numerical study of a Sierpinski triangle to investigate if it has properties that may be relevant for SHM. The numerical simulations were conducted in Abaqus. The structure was modeled with Timoshenko beam elements, and each beam in the lattice was discretized into 10 elements after a mesh convergence analysis. Since we are only interested in the out-of-plane behavior of the structure, in-plane degrees of freedom were constrained.

Comparison with triangular lattice

We conducted an eigenfrequency simulation to analyze the nature of the spectrum. We used the Lanczos solver to calculate the eigenfrequencies and the eigenvectors related to the first 4000 modes. The analysis involved a regular triangular lattice with 32 beams per edge, and a Sierpinski triangle with the same edge size. The geometry and spectra are shown in Fig. 2.

The comparison showed that the regular triangular beam lattice is characterized by a single bandgap at the characteristic frequency of the smallest beam in the lattice. In contrast, the Sierpinski triangle shows multiple bandgaps, each associated with the size of a generation. Although the appearance of additional gaps is interesting and corroboratory of similar phenomena in other systems, the most unique feature of the presented spectrum is found in how these gaps are organized. Fig. 3 shows the scale invariance of the spectral bands of a 5-generation Sierpinski triangle by zooming into the spectrum at intervals spaced in the modal index by factors of 3. This factor of 3 is the base unit of the exponential left-hand side of Eq. 1, and is the factor that grows the system to its consecutive hierarchical length scales. We see that in each zoom the structure of the spectrum is retained, and we find that another gap takes the place of the principal (highest frequency) gap in this relative frame.

Sensitivity to damage

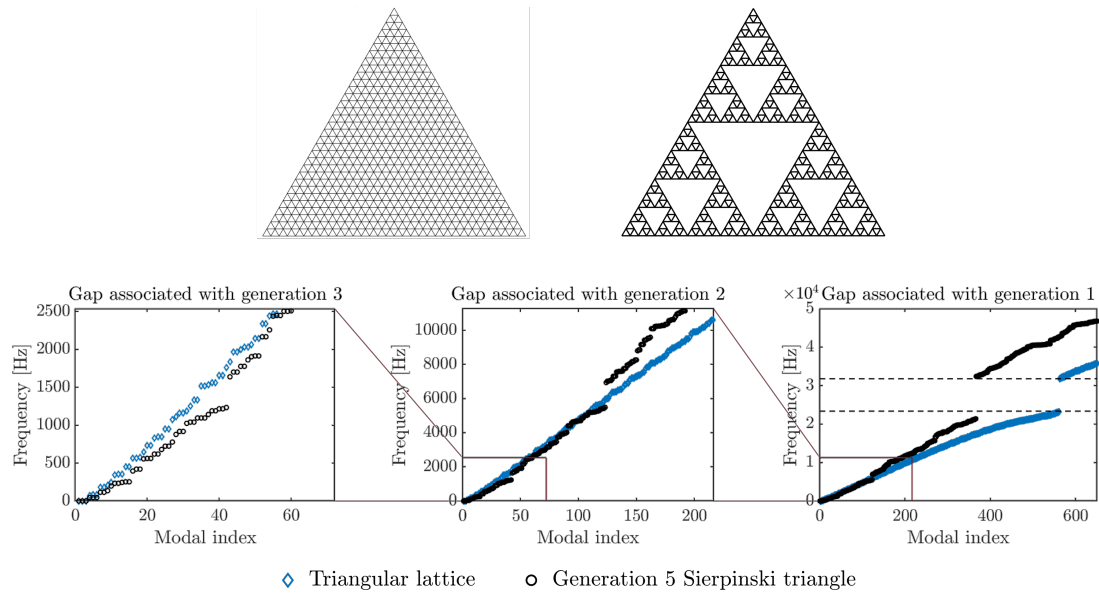


Figure 2. Triangular beam lattice and Sierpinski triangle. The comparison of the spectrum of the two structures shows that the regular lattice is characterized by a single bandgap, while the Sierpinski triangle shows multiple bandgaps across different scales, each associated to a different generation.

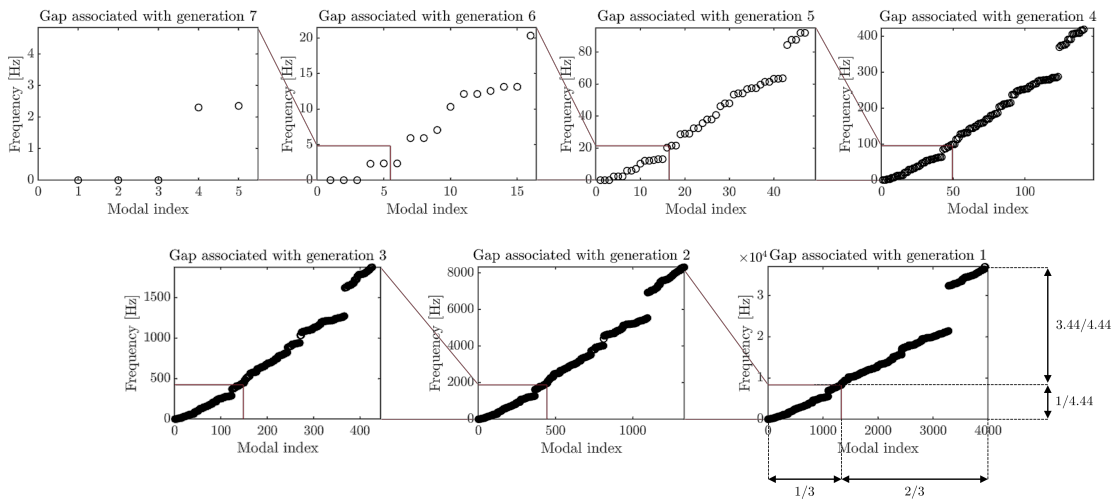


Figure 3. Self-similar spectrum of a 7-generation Sierpinski triangle. We see that in each zoom, the structure of the spectrum is retained, and we find another gap takes the place of the principle gap in this relative frame.

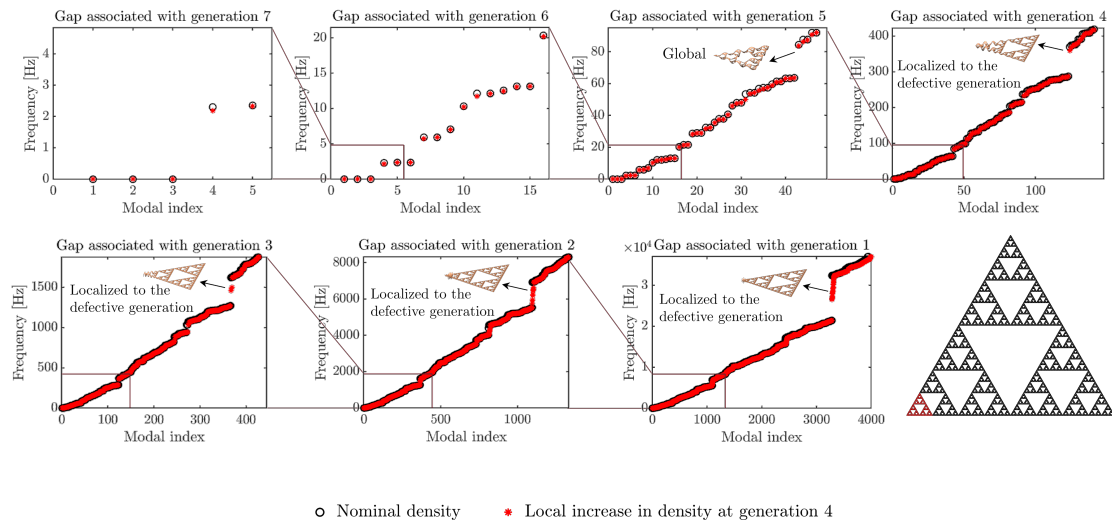


Figure 4. Spectrum of a 7-generation Sierpinski triangle containing a defective 4-generation triangle positioned at the bottom left corner. The bandgaps associated with the generational orders included in the defective region are populated by damage modes, and each mode remains spatially confined to the corresponding defective generation.

The spectrum of the Sierpinski triangle was further studied by introducing defects in the lattice. Defects were introduced as local variation of the density of the defective generation.

The results shown in Fig. 4 refer to the case of a defective 4-generation triangle at the bottom left corner of a 7-generation Sierpinski triangle. The simulation showed that the bandgaps associated with the generations included in the defective triangle were populated by damage modes localized to the corresponding defective generation. Instead, lower-frequency bandgaps associated with generations larger in size than the defect were not populated by such modes. This finding brings evidence that a fractal sensor, or a fractal self-sensing structure, can be used to diagnose damage simply by performing modal analysis. Furthermore, the self-similar properties of the architecture allow for multi-scale damage sensitivity.

CONCLUDING REMARKS

This work has explored the potential of fractal geometries, specifically the Sierpinski triangle, for structural health monitoring (SHM) systems.

Numerical simulations have been conducted to bring evidence that fractal sensors or self-sensing structures possess unique multiscale spectral characteristics with multiple frequency bandgaps, each associated with a generation of the fractal. Furthermore, the hierarchical structure of the Sierpinski triangle inherently provides sensitivity to multi-scale damage. Our studies have also verified that defects, simulated as local density variations, distinctly affect specific frequency bandgaps corresponding to the scale of the defective generations. The bandgaps associated with larger scales remained unaffected, clearly indicating the damage localization capacity of the proposed architecture.

The damage sensitivity and multiscale response of fractal structures highlight the

potential of such architectures for alternative and innovative SHM solutions which can deduce both mass and size of an attachment or defect, eventually eliminating the need for external sensors. Future research will explore alternative fractal patterns to identify additional properties that can be leveraged for SHM, and will also focus on the experimental validation of numerical predictions. Two complementary approaches will be pursued: self-sensing structures incorporating fractal patterns will be fabricated by waterjet cutting metal plates, while miniaturized sensors will be produced using higher-resolution techniques, such as direct ink writing [17].

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