High-Fidelity Analytical Approaches for Damage Detection and Monitoring in Curved Composite Beams

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Structural analysis and structural health monitoring (SHM) are essential procedures for ensuring the safety and operational efficiency of aerostructures, particularly composite laminates with complex geometries subjected to static or dynamic loadings. These structures, widely used in aerospace applications such as aircraft nacelles, are often susceptible to impact induced damages which may compromise the performance of structural parts leading to undesirable outcomes. Consequently, characterising the effect of damage on the performance of composite aerostructures becomes inevitable. To mitigate the time-demanding in-service assessment of aerostructure components, this research proposes a high-fidelity analytical-based SHM model for damage detection in curved composite beams. Firstly, a one-dimensional (1D) analytical model which analyses the mechanical response of curved composite beams under arbitrary boundary and loading conditions is developed. The 1D model adopts the kinematical descriptions of refined Timoshenko beam theory enhanced with deterministic continuum damage mechanics principles to functionally achieve stiffness degradation that typifies bulk damages. The ensuing equilibrium equations are then analytically discretised leading to a global system with which the natural and resonant frequencies of damaged structures are simulated. Calibration of the 1D model's capacity through comparison with 3D Finite Element (FE) solutions and literature benchmarks indicate high-fidelity and significant computational savings in degrees of freedom and runtime. The 1D model is subsequently integrated with swarm intelligence-based optimization algorithms to detect damage in composite laminates. A demonstration of the capability of the analytical-based SHM approaches reveals that rapid damage detection and quantification across various material configurations and boundary conditions can be achieved with satisfactory accuracy emphasising the robustness of the SHM scheme. In view of this outcome, the proposed SHM model offers great potential for real-time SHM applications while contributing to reduced maintenance costs and enhanced structural reliability of composite aerostructures.

Keywords: SHM, 1D analytical model, damage detection, swarm intelligence-based optimization.

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1. Introduction

The rapid development of manufacturing and material technologies has contributed to the increased application of composite materials that offer high strength-to-weight ratio and good vibration damping properties to meet diverse industrial needs [1]. In the context of composite structures, curved beams are important components of many lightweight structures, for example, helicopter blades, wind turbine blades, aircraft wings, due to improved weight and force distribution that enhance overall structural stability and the dynamic response [2, 3]. However, composite structures are characterised by complex damage mechanisms which limit their potential application-wise. Such damage scenarios may arise from manufacturing defects or service-related applications which eventually render the composite structures unsafe for prolong use. To mitigate this drawback, it is crucial to set up damage detection system that assesses the integrity of the structure, thus preventing safety problems and minimize maintenance costs. Traditionally, visual inspection, acoustics, infrared thermography and ultrasonic waves are commonly employed methods to identify damages in engineering structures. However, these methods may suffer setbacks when the damage area is not visible for inspection which is often the case for composite materials which may experience phenomena such as delaminations, matrix cracking, fibre breakage [4]. A practical remedy for this is the vibration-based damage identification method which makes use of vibration measurement at any location in the structure to detect and predict the extent of damage which can compromise the functional as well structural integrity of the composites.

From structural health monitoring (SHM) standpoint, changes in vibration-based indicators such as natural frequencies and mode shapes can be used to detect damage in structures through optimization of an objective function which maps the search space to the function space (constructed via measured or simulated data) [5-7]. In this context, Jebieshia et al [4] exploited the unified particle swarm optimization (UPSO) technique to conduct a frequency-based assessment of plate and beam composite members based on a finite element (FE) formulation and continuum damage mechanics. Within this integrated framework, anisotropic damage mechanics principle was invoked based on structural stress reduction factor in different directions according to the fibre orientations [8]. The main advantage of this approach is that the principal damage variables can be conveniently defined and parametrically incorporated into element-based formulation such as finite element method (FEM) yet such FE-based approach can incur high computational cost due to low-order mesh refinement (i.e., *h*-refinement), thus impairing SHM-related maintenance. An alternative approach to overcome this bottleneck is analytical approach which can model the structural responses of the laminated composites while avoiding the numerical complexities of the so-called *h*-refinement associated with FEM. In the field of analytical discretisation, the application of differential quadrature method (DQM) offers robust solutions for engineering systems beyond the limitations of analytical closed-form solutions and with superior efficiency over FE computations (see [9,10]). Thus, DQM guarantees higher-order accuracy for systems with diverse boundaries, material distributions and loading conditions.

In an attempt to enhance SHM operations and ease decision making, this study proposes an analytical-based approaches for damage detection and monitoring in curved composite beams. The framework integrates an analytical structural model with swarm intelligence-based optimization techniques including particle swarm optimization (PSO) [11], UPSO [12] and butterfly optimization algorithm (BOA) [13] to conduct efficient damage detection of curved composite beams. The structural model is kinematically described by the Timoshenko beam theory [14] enhanced with anisotropic damage mechanics constitutive laws [8] while DQM [8] is employed for the analytical solution.

2. Theoretical formulation

The displacement and strain fields of a curved beam with radius R, assuming small deformation, are defined as [15] (see Figure 1)

$$u(\theta,y,z,t) = u_0(\theta,y,t) + z\varphi_0(\theta,y,t), \quad [1a]$$

$$w(\theta,y,z,t) = w_0(\theta,y,t), \quad [1b]$$

$$\varepsilon_{\theta\theta} = \frac{1}{R(1+z/R)}\left[\frac{\partial u_0}{\partial\theta} + z\frac{\partial\varphi_0}{\partial\theta} + w_0\right], \quad [2a]$$

$$\gamma_{\theta z} = \frac{1}{R(1+z/R)}\left[\frac{\partial w_0}{\partial\theta} + R\varphi_0 - u_0\right], \quad [2b]$$

where u and w are, respectively, the in-plane and transverse displacements whereas the mid-plane displacements terms (at $z = 0$), u_0, φ_0 and w_0 represent the in-plane stretching along the curvilinear coordinate (θ), rotation about transverse normal along θ and transverse deflection in the z-coordinate respectively. In addition, $\varepsilon_{\theta\theta}$ denotes the normal strain along θ-direction while $\gamma_{\theta z}$ represents the transverse shear strain in the θ-z plane. All these parameters are dependent on time t.

The stress-strain relations defining the constitutive behaviour of the k^{th} layer of a composite laminate, assuming plane-stress conditions, are given by [16]

$$\sigma_{\theta\theta}^k = \hat{Q}_{11}^k\varepsilon_{\theta\theta}^k, \quad [3a]$$

$$\tau_{\theta z}^k = k_s\hat{Q}_{55}^k\gamma_{\theta z}^k, \quad [3b]$$

where $\sigma_{\theta\theta}^k$ and $\tau_{\theta z}^k$ are the normal and transverse shear stresses, while $\hat{Q}_{11}^k$ and $\hat{Q}_{55}^k$ are plane-stress related effective 3D material coefficients defined in [16,17]. Finally, k_s is the shear correction factor.

2.1 Dynamic equilibrium equations

The dynamic equilibrium equations are derived according to Hamilton's principle which leads to the strong-form equations [16,17]

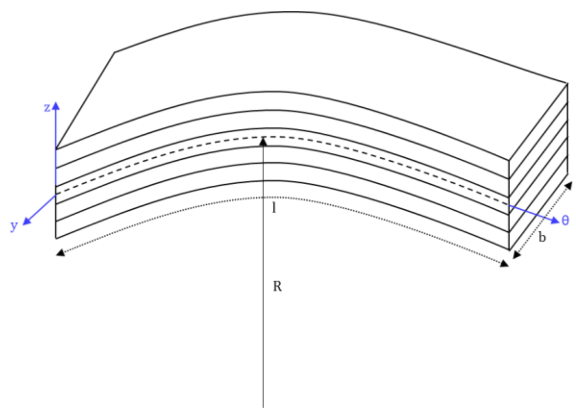

Figure 1. laminated curved beam frame of reference.

$$\frac{\partial N_{\theta\theta}}{\partial\theta} + Q = I_0\ddot{u}_0 + I_1\ddot{\varphi}, \quad [4a]$$

$$\frac{\partial Q}{\partial\theta} - N_{\theta\theta} + F_z(t) = I_0\ddot{w}_0, \quad [4b]$$

$$\frac{\partial M_{\theta\theta}}{\partial\theta} - QR = I_1\ddot{u}_0 + I_2\ddot{\varphi}\ , \quad [4c]$$

where $F_z(t) = R(1 + h/2R)q(t)$. Substituting the displacements in Equations (1-2) into Equation (4) while setting the force (F_z) to zero for a free vibration case leads to the displacement-based compact form

$$\boldsymbol{K}\boldsymbol{u} + \boldsymbol{M}\ddot{\boldsymbol{u}} = \boldsymbol{0} \quad [5]$$

Assuming the solution takes the form $\boldsymbol{u}(t) = \boldsymbol{\mathcal{U}}e^{i\Omega t}$, Equation (5) can be re-constructed in the eigenvalue form

$$(\overline{\boldsymbol{K}} + \Omega^2\overline{\boldsymbol{M}})\boldsymbol{\mathcal{U}} = \boldsymbol{0} \quad [6]$$

where $\boldsymbol{\mathcal{U}}$ are the modal solutions while Ω is the natural frequency. Finally, the differential quadrature method is applied to Equation (6) through the approximation

$$\boldsymbol{\mathcal{U}}(\theta) = \sum_{i=1}^{N} l_i(\theta)\tilde{\boldsymbol{\mathcal{U}}}_i \quad [7]$$

where l_i are Lagrange polynomials specified on a Chebyshev-Gauss-Lobatto grid. Substituting Equation (7) into Equations (6) gives the free vibration solution of the system for different geometric, material, lamination and boundary configurations.

2.2. Damage formulation

Structural damage is captured according to continuum damage mechanics constitutive laws [18] whereby localised stiffness degradation is induced through an anisotropic damage coefficients $\Gamma_i \in [0\ 1]$ (for $i = 1,2,3$) which account for damage severity ranging from 0 for a pristine structure without damage and 1 for a fully damaged structure. With the the damage coefficients defined, the material coefficients i.e, E_1, E_2, E_3, G_{12}, G_{13}, G_{23}, v_{12}, v_{13}, and v_{23}, become functions of the damage parameter Γ, and are computed according to the formulae detailed in [18]. Consequently, the effective material stiffness coefficients of the damaged structure are re-expressed as

$$\sigma_{\theta\theta}^{k} = \hat{Q}_{11}^{k}(\boldsymbol{\Gamma})\varepsilon_{\theta\theta}^{k}, \quad [8a]$$

$$\tau_{\theta z}^{k} = k_s \hat{Q}_{55}^{k}(\boldsymbol{\Gamma})\gamma_{\theta z}^{k}, \quad [8b]$$

where $\boldsymbol{\Gamma} = [\Gamma_1 \quad \Gamma_2 \quad \Gamma_3]^{\mathrm{T}}$ is the vector of damage parameters.

To simulate a bulk damage scenario in the beam, the damage parameters, i.e., Γ_i, is idealised as functions of the beam axis θ, i.e., $\boldsymbol{\Gamma}(\theta)$. In this context, damage localisation along the beam spanning from θ_a to θ_b is expressed as

$$\begin{cases} 0 < \Gamma_i \leq 1 & \theta_a < \theta < \theta_b \\ \Gamma_i = 0 & \theta_b \geq \theta \geq \theta_a \end{cases}, \quad [9]$$

where θ_a and θ_b denote the start and end of the damage area (see Figure 2).
To preserve the continuity of the strong-form equilibrium equations (i.e., Equations (4a-4c)), Equation (9) is adaptively implemented based on a continuous representation assumed as

$$\boldsymbol{\Gamma}(\theta) = \boldsymbol{\Gamma} \sum_{n=\theta_a}^{\theta_b} \exp^{-\left[\frac{(\theta-\theta_n)^2}{2\sigma^2}\right]} \quad [10]$$

where σ is the damage spread. Finally, in the presence of damage, the parameters Γ_i ($i = 1,2,3$) can be tuned to determine the natural frequencies of the damaged beam.

3. Swarm intelligence-based optimization algorithms

Swarm intelligence-based optimization are generally intended to optimize a population within a search space with respect to a given meausure of quality. The search is typically characterised by movements of particles which are guided towards the best solution (in an exploitative global search) and directly influenced by local best solution (in an explorative local search). In the context of damage identification in this study, swarm intelligence-based optimization algorithms have been considered to detect and determine the level of damage in 3D curved composite beams whose natural frequencies (ω) are the field inputs generated from 3D FE simulation. To realize this aim, the suitability of the search solution to achieve global optimum is determined through the fitness function, F

$$F = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\omega_i^{3D} - \omega_i^{1D})^2} \qquad [11]$$

where ω_i^{3D} and ω_i^{1D} are the 3D FE-simulated and 1D-simulated natural frequencies of the damaged structure, respectively, and n is the number of input frequencies considered. In this context, three frameworks, namely PSO [11], UPSO [12] and BOA [13] have been implemented.

4. Numerical examples

This section illustrates the validity of the analytical-based SHM procedures through new benchmarks involving two semicircular 3D curved composite beam configurations, each configuration subjected to clamped-clamped (CC) and clamped-free (CF) boundaries. The beam definitions are outlined in Table I whereas the material of the beam is a carbon fibre reinforced polymer (CFRP) with the properties $E_1 = 144.8$ GPa, $E_2 = E_3 = 9.65$ GPa, $G_{12} = G_{13} = 4.14$ GPa, $G_{23} = 3.45$ GPa, $v_{12} = 0.3$, $v_{13} = 0.63$, $v_{23} = 0.4$ and $\rho = 1389.23$ kg/m^3 [4].

The 3D curved beams designated as Beam-1 and Beam-2 in Table I are induced with a bulk damage that is homogeneously idealised within an area (centred at $\theta_c = 0.5L$) covering 10% of the beam's length and having degraded material coefficients computed by setting $\Gamma_1 = \Gamma_2 = \Gamma_3 = 0.2$ (see Figure 2). The 'true' natural frequencies are simulated with C3D20R solid elements in Abaqus and are used as input parameters to the optimization codes. Employing the 1D beam model, the optimization codes randomly initialize the damage coefficients in the 1D beam as $\Gamma_i \in [0, 0.9]$ and iteratively minimizes the fitness function to determine the best values of Γ which match the input frequencies of the 3D beam.

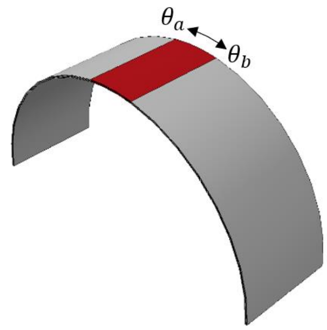

Figure 2. Damaged curved beam representation showing damaged section (in red colour).

4.1 Damage detection analysis of curved composite beams

Table II shows that PSO, UPSO and BOA algorithms fail to accurately predict the level of damage i.e. Γ, in the CC curved beam with one input frequency. This outcome is attributed to many local solutions when the input data is insufficient for the algorithms to distinguish between local equilibriums and the global equilibrium. On increasing the number of input frequencies to two and above, all the agorithms converged to the correct value of Γ by over 90% margin of accuracy indicating high-fidelity and numerical stability. It is observed that the three algorithms implemented exhibited the same level of accuracy. In the case of CF beam, three input frequencies are inadequate to accurately estimate damage severity (Γ) in the 3D beam (see Table III). In contrast to CC beam, CF beam experiences 3D free edge effect concerning which the 1D analytical model is kinematically inadequate to capture. This limitation leads to the presence of many local solutions characterised by frequencies that are not captured by the 1D analytical-based SHM procedure. Nonetheless, by increasing the number of input frequencies to four, a good prediction of the damage level can be realized, according to Table III. Considering that the 1D analytical-based SHM procedure is computationally cheap, these outcomes potentially simplify damage identification in 3D structures, hence contribute to low-cost maintenance.

5. Conclusions

This research proposes analytical-based SHM procedures for curved composite beams using a one-dimensional (1D) analytical structural model integrated with swarm intelligence-based optimization techniques. The 1D model is kinematically described by the Timoshenko beam theory and an adaptive continuum damage mechanics constitutive laws while employing the differential quadrature method to obtain the free vibration solution analytically. The 1D model is subsequently integrated with particle swarm optimization, unified particle swarm optimization and butterfly optimization algorithms to conduct damage detection and quantification in a damaged 3D curved composite beams whose natural frequencies are computed from 3D FE simulations. From the benchmark examples, the optimization algorithms demonstrate excellent potential for efficient damage detection and quantification in the curved composite beams with different boundary conditions. Remarkably, more input frequencies are required for damage detection in clamped-free beams than clamped-clamped ones due to the kinematical complexities introduced by 3D free edge effects. Based on these outcomes, the analytical-based SHM aproaches can potentially contribute to real-time SHM operations while mitigating high maintenance costs. Future study will consider multi-parameter optimization encompassing damage localization and quantification to practically aid prognosis of engineering structures in service.

Table I. Beam definitions for numerical tests.

Beam	Material	Stacking sequence	Geometry	Boundary condition
Beam-1	CFRP	$[0^o/90^o/90^o/0^o]$	$L = 5.8$ m, $R = 5L/\pi$ m $h = 0.025$ m $b = 1.2$ m Damage centre, $\theta_c = 0.5L$, Damage centre, $\gamma = 10\%$	CC
Beam-2	CFRP	$[0^o/90^o/90^o/0^o]$	$L = 5.8$ m, $R = 5L/\pi$ m $h = 0.025$ m $b = 1.2$ m Damage centre, $\theta_c = 0.5L$, Damage centre, $\gamma = 10\%$	CF

Table II. Damage severity (Γ) prediction for Beam-1.

Total no. of input modes	ω (Hz) (3D-FE simulated for $\Gamma = 0.2$)	PSO (% Error)	UPSO (% Error)	BOA (% Error)
1	13.94	0.30 (50%)	0.31 (55%)	0.31 (55%)
2	29.47	0.22 (10%)	0.20 (0%)	0.20 (0%)
3	56.32	0.20 (0%)	0.21 (5%)	0.21 (5%)
4	83.88	0.20 (0%)	0.20 (0%)	0.22 (10%)

Table III. Damage severity (Γ) prediction for Beam-2.

Total no. of input modes	ω (Hz) (3D-FE simulated for $\Gamma = 0.2$)	PSO (% Error)	UPSO (% Error)	BOA (% Error)
1	1.37	0.12 (40%)	0.12 (40%)	0.12 (40%)
2	4.19	0.11 (45%)	0.11 (45%)	0.11 (45%)
3	14.95	0.13 (35%)	0.13 (35%)	0.13 (35%)
4	32.22	0.20 (0%)	0.20 (0%)	0.20 (0%)

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