Experimental and Numerical Investigations on the Acoustoelastic Effect on Lamb Waves in Aluminum

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ABSTRACT

Damage detection in thin-walled components is a common application of guided ultrasonic waves, such as Lamb waves, in today's structural health monitoring. However, the method requires that external or manufacturing influences such as residual stresses, pre-stresses or stresses due to temperature are taken into account. Therefore, this work focuses on the field of acoustoelasticity and compares the influences of pre-stress on the phase velocities of Lamb waves in numerical studies with experimentally determined results. For this purpose, previous experimental and numerical investigations of the authors are extended by expanding the frequency ranges and load steps considered. The measurements are carried out using laser vibrometry to capture the velocity field of the Lamb waves in aluminum specimens subjected to uniaxial pre-stresses covering the elastic range of the material. The experimental results are thereby characterized by the use of a 2d Fourier transform to evaluate the phase velocity over large frequency ranges. The numerical data are generated using a two-step FE model that combines a nonlinear static and a linear eigenvalue computation to determine the dispersion behavior of Lamb waves subjected to pre-stress. The subsequent comparison of numerical and experimental investigations focuses on the verification of the Neo-Hookean and Murnaghan hyper-elastic material models by comparing the changes in phase velocities induced by the pre-stress. For this purpose, the numerically obtained results for different material models are compared with experimental data. The results of these investigations provide a clear indication of the suitability of the material models used here to represent the acoustoelastic effect in Lamb waves.

INTRODUCTION

The use of Lamb waves to detect damage in thin-walled components is a common application in the field of today's structural health monitoring [1, 2]. This applications requires comprehensive knowledge of the propagation behavior of the waves in the component to be monitored. For this reason, this work deals with the influence of pre-stress states on the phase velocity of Lamb waves. The objective of this work is to extend

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the results in Barth et al. [3] and thus enhance the validation of numerical simulations against experimental results. For this purpose, the influences of uniaxial pre-stress states in the propagation direction on the fundamental Lamb waves modes are investigated.

The field of acoustoelasticity deals with the influences of stress states on the propagation behavior of elastic waves [4, 5]. Fundamental investigations in the field of acoustoelasticity go back to geophysical problems. There, Biot [6, 7] developed a theory for the analysis of elastic waves under the influence of initial stress. Following this, Thurston and Brugger [8], Hughes and Kelly [9], and Toupin and Bernstein [10] studied the determination of third-order elastic constants by experiments for use in the constitutive theory of Murnaghan [11]. Among others Crecraft [12], Hsu [13], and Blinka and Sachse [14] used longitudinal and shear waves to determine elastic stress states in solids. For a review of the theory of acoustoelasticity and also acoustoplasticity with emphasis on hyper-elastic anisotropic materials, the reader is referred to Pao and Gamer [5] and the sources therein. Essential properties of the material models suitable to represent pre- and residual stresses are discussed in the work of Man et al. [15] and Shams et al. [16] and references therein.

For Lamb waves, a corresponding theoretical study can be found in the work of Husson [17] and Qu and Liu [4]. Using the semi-analytical finite element method Wilcox [18] and Loveday [19] present numerical investigations on this topic. However, literature dealing with the validation of numerical and theoretical results through experimental data are rare. Gandhi et. al. [20] investigate the acoustoelastic effect by measuring wave velocities of Lamb waves in biaxially stressed plates and compare the results with analytical results. With emphasis of numerical multi-physical simulations, regarding piezoelectric transducers, Qiu et al. [21] investigate the influence to the wave propagation for uniaxially stressed plates. In the work of Barth et al. [3] phase velocities of Lamb waves in uniaxially stressed plates are experimentally measured and compared with numerical results. Here, the emphasis is on investigating the effect over a large frequency range and verify the suitability of different hyper-elastic material models.

The results in Barth et al. [3] provide a first insight into the suitability of selected hyper-elastic material models for modeling the effect of pre-stress states on the propagation properties of Lamb waves. In this work, these findings are to be extended and validated by means of more detailed experimental and numerical investigations. For this purpose, the influence of this effect is investigated, especially in the frequency ranges that are particularly suitable for a comparison of experimental and numerical results. In addition, the consideration of the influence is validated on the basis of multiple stress states.

The structure of the presented work is as follows. First, a brief insight into the fundamentals of acoustoelasticity and the material laws used is given. Furthermore, the basics of the experimental and numerical investigations used are presented. The experimental and numerical results are then examined and compared with respect to the influence of pre-stress on the phase velocity of the two fundamental Lamb wave modes. In addition, the influence of the constitutive laws, the Neo-Hooke and the Murnaghan model, is analyzed. Finally, based on these results, a conclusion is given on the suitability of the hyper-elastic material models considered here to represent the influence of pre-stress on the propagation behavior of Lamb waves.

ACOUSTOELASTICITY

Acoustoelasticity deals with the influence of stress states on the propagation behavior of waves in elastic materials. In this work, the total deformation of an elastic body is decomposed into two parts, one due to the pre-stress and the other due to the guided wave. Mathematically, this implies a multiplicative decomposition of the deformation gradient $\mathbf{F}^F = \mathbf{F}^D \cdot \mathbf{F}^S$, introducing an intermediate configuration in addition to the reference and current configurations. The resulting balance of linear momentum reads

$$\mathrm{Div}\left(\mathrm{Grad}\,\mathbf{u}^D \cdot \mathbf{S}^S + \mathbf{F}^F \cdot \mathbf{S}^D\right) = \rho_0 \frac{\partial^2 \mathbf{u}^D}{\partial t^2} \quad . \tag{1}$$

Therein $\mathbf{u}^D$ corresponds to the dynamic motion of the wave propagation, ρ_0 to the density in the reference configuration, $\mathbf{S}^S$ and $\mathbf{S}^D$ to the second Piola-Kirchhoff stress tensor due the static and dynamic loading, respectively, and $\mathbf{F}^F$ to the deformation gradient between reference and current configurations, which is defined as

$$\mathbf{F}^F = \mathbf{I} + \mathrm{Grad}\,\mathbf{u}^F = \mathbf{I} + \mathrm{Grad}\,\mathbf{u}^S + \mathrm{Grad}\,\mathbf{u}^D \quad . \tag{2}$$

It is to be noted that all mathematical operations shown refer to the reference configuration. A more detailed description of these relationships can be found e.g. in [3, 22]. In addition to the geometric non-linearity, hyper-elastic material models are used to represent the non-linear material behavior. Therefore, in addition to the Murnaghan [11] material model, often used in the literature for acoustoelastic analysis, see e.g. [23, 24], a compressible Neo-Hookean material model is used. The corresponding strain-energy density function for an isotropic material according to Murnaghan reads

$$\begin{aligned}\Psi^{\mathrm{Mur}}(\mathbf{E}) =& \frac{1}{2}(\lambda + 2\mu)\,[\mathrm{tr}(\mathbf{E})]^2 - 2\mu\,\mathrm{tr}(\mathbf{E}^2) \\ &+ \frac{1}{3}(l + 2m)\,[\mathrm{tr}(\mathbf{E})]^3 - 2m\,\mathrm{tr}(\mathbf{E})\,\mathrm{tr}(\mathbf{E}^2) + n\,\mathrm{tr}(\mathbf{E}^3) \quad .\end{aligned} \tag{3}$$

Here, $\mathbf{E}$ is the Green-Lagrangian strain tensor and μ and λ are the Lamé-constants known from the linear theory of elasticity. The so called third-order elastic constants $l, m,$ and n represent the nonlinear part of the material behavior. In comparison, the strain-energy density function for the Neo-Hookean material model according to [25] reads

$$\Psi^{\mathrm{Neo}}(\mathbf{C}) = \frac{1}{2}\lambda\,[\ln(J)]^2 - \mu\ln(J) + \frac{1}{2}\mu\,[\mathrm{tr}(\mathbf{C}) - 3] \quad , \tag{4}$$

which is a function of the right Cauchy-Green tensor $\mathbf{C}$ and the Jacobian $J = \det \mathbf{F}$. Like Eq. (3), this nonlinear material model also uses the Lamé parameters μ and λ but it does not require any higher order constants, which are difficult to identify.

EXPERIMENTAL INVESTIGATIONS

The aim of the experimental investigations in this work is to determine the dispersion relation of Lamb waves over large frequency ranges with high accuracy. For this purpose, the method shown in [26, 27] is used. It uses specially tuned multifrequency excitation signals generated by piezo actuators on the surface of the specimens. Laser vibrometry measures the velocities of material points along a path on the surface of specimens during wave propagation. The data evaluation is based on a 2d-DFT to evaluate the dispersion relations in the wavenumber-frequency domain, see e. g. [28].

The results presented in this work are based on measurements in aluminum strips (EN AW-5754) which are subjected to pre-stress using a tensile testing machine. The length and width of the specimens are $1100\,\mathrm{mm}$ and $110\,\mathrm{mm}$, respectively. The thickness depends on the frequency or wavenumber range to be investigated. For the measurements of the S_0-mode a thickness of $2\,\mathrm{mm}$ and for the A_0-mode a thickness of $0.5\,\mathrm{mm}$ is used. The selection of the frequency ranges considered is determined in advance on the basis of numerical investigations and is aimed at ranges in which a particularly well measurable influence of the pre-stress on the wave propagation is to be expected.

NUMERICAL INVESTIGATIONS

The dispersion relation of Lamb waves can be computed numerically by performing a linear eigenvalue analysis of a one-dimensional model that represents the thickness direction of the wave guide, see e.g. [29,30]. Via slight adaptations, this approach is able to compute the dispersion relations of wave guides under pre-stress, see e.g. [3, 18, 19].

In order to perform these investigations, the one-dimensional model is extended to a two-dimensional model, which additionally represents a fraction of the wave guide in the propagation direction of the waves. Therefore, the used model includes the whole thickness of the wave guide, but only a $0.1\,\mathrm{mm}$ long part in the propagation direction of the waves. This extension allows a two-stage simulation in which at first a static analysis is performed in order to induce a pre-stress in the propagation direction of the waves. Subsequently, the results of this first step are used as a basis for the linear eigenvalue analysis, which calculates the dispersion relation of the Lamb waves under the influence of pre-stress. In order to comply with the boundary conditions for the propagation of Lamb waves, periodic boundary conditions at the edges in the direction of propagation are specified in addition to stress-free boundary conditions at the top and bottom of the model. For discretization, second-order Lagrange finite elements are used.

The hyper-elastic material models in Eqs. (3) and (4) are used to investigate the effect of pre-stress. The required material parameters are given in Tab. I (a) and (b).

TABLE I. MATERIAL PARAMETERS.

(a) Material constants for EN AW-5754.

Parameter	Value	Unit
ρ_0	2700	$\mathrm{kg/m^3}$
μ	25.6	GPa
λ	49.6	GPa

(b) Third-order elastic constants for EN AW-7075 [31].

Parameter	Value	Unit
l	-252.2	GPa
m	-325.0	GPa
n	-351.2	GPa

RESULTS AND DISCUSSION

In the following, the results of the experimental and numerical investigations are compared. For this purpose, the differences in phase velocities between various pre-stress states are investigated. The following applies for the differences in phase velocities

$$\Delta c_p = c_p^{LS} - c_p^{Ref} \quad . \tag{5}$$

Here, c_p^{LS} corresponds to the phase velocity of the stress state considered and c_p^{Ref} to the phase velocity in the reference stress state which is taken as $10\,\mathrm{MPa}$. The use of a reference state with non-zero pre-stress leads to a compensation of influences from manufacturing-induced residual stress states and induced stress states from the clamping process in the tensile testing machine.

Fig. 1 presents the results for the difference in phase velocities resulting from a pre-stress difference of $10\,\mathrm{MPa}$, $50\,\mathrm{MPa}$ and $90\,\mathrm{MPa}$ for the two fundamental wave modes. Fig. 1 (a) - (c) shows the results for the S_0-mode for a frequency-thickness product between $1.5\,\mathrm{MHz\,mm}$ and $3.5\,\mathrm{MHz\,mm}$. This range was chosen because here both material models used in the numerical models predict a large change in the influence of pre-stress on the phase velocity. Although the behavior of both material models in this range shows similarities, a significant difference can be observed. The Murnaghan material model predicts a decrease in phase velocity for the entire range, while the Neo-Hookean material model shows a change from a decrease to an increase in phase velocity. This change of influence is also clearly visible in the experimental data. Therefore, based on the comparison with the experimental data, it can be concluded that the Neo-Hookean material model provides a more accurate representation of the qualitative course of the influence on the phase velocity.

Fig. 1 (d) - (f) depicts the results for the A_0-mode for a frequency-thickness product up to $0.3\,\mathrm{MHz\,mm}$. Identical data are presented in Fig. 1 (g) - (i), however, the lower limit of the frequency-thickness product is moved to $0.03\,\mathrm{MHz\,mm}$ to allow a better representation at higher ranges. Similar to the S_0-mode, this frequency-thickness product was selected based on the numerical simulations. However, this range is not only characterized by a strong change in the influence but also provides the effectively highest influence of the pre-stress on the phase velocity. The comparison of the two material models used in the numerical simulations shows that they provide almost identical results in the very low range of the frequency-thickness product. This is due to the primary influence of the geometric nonlinearity in this range. In the higher frequency-thickness range, the influence of the material models is more significant and a difference in influence for the two material models becomes visible. As in the S_0 mode, the most striking feature is the change in influence on the phase velocities that occurs in the Murnaghan material model but not in the Neo-Hookean material model. Again, the comparison with the experimental data in Fig. 1 (g) - (i) shows that the Neo-Hookean material model predicts the influence of the pre-stress on the wave propagation more accurately.

It should be noted here that an adjustment of the third-order elastic constants from Tab. I (b) could improve the results for the Murnaghan material model. However, their experimental determination is laborious and difficult, and very few literature references provide these constants. Furthermore, the use of a model with large uncertainties in the material parameters cannot be recommended, especially since the Neo-Hookean model can represent the influence without the uncertainties of these constants.

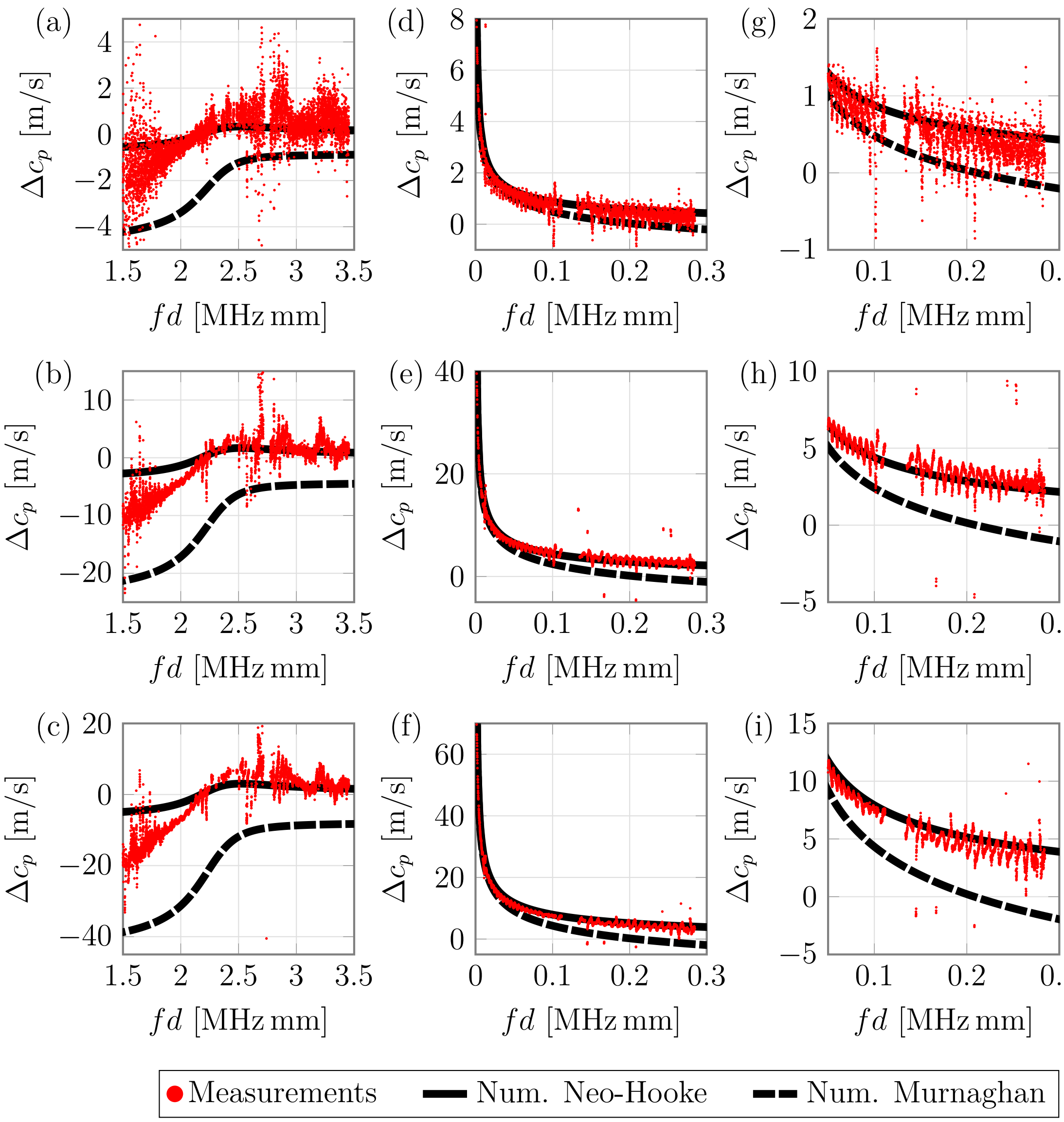

Figure 1. Difference in the phase velocity due to pre-stress. Comparison of numerical and experimental results. (a)-(c): S_0-Mode for pre-stress differences of 10 MPa, 50 MPa and 90 MPa. (d)-(f): A_0-Mode for pre-stress differences of 10 MPa, 50 MPa, and 90 MPa. (g)-(i): like (d)-(f), but on a different scale.

CONCLUDING REMARKS

In this work, the influence of pre-stress on the propagation of Lamb waves is investigated. Based on [3], the material behavior of two different nonlinear material models, namely the Neo-Hookean and the Murnaghan model, have been considered. For this purpose, the known results from [3] were both extended to specifically selected frequency-thickness products and validated using various pre-stress states. The results of the Neo-Hookean material model are in very good agreement with the experimental data, while the Murnaghan model shows higher deviations from the experimental results. In addition, the Neo-Hookean material model has the advantage of not requiring higher order elastic constants, whereas the use of the Murnaghan material model is less appropriate since these constants are complicated to determine and hardly available in the literature. Therefore, the presented results further support the use of the Neo-Hookean material model over the Murnaghan model, which is commonly used in the literature for numerical investigations regarding the acoustoelastic effect.

The results of this work form a solid basis for the validation of the various modeling approaches on this topic for the material used here. In the further course of this work, based on the findings presented here, the influence of pre-stress states in fiber-reinforced materials will be investigated. In combination with these results, fundamental insights can then be gained for the modeling of manufacturing-induced residual stresses in composite materials.

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