On Metrics for Information Value Quantification

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ABSTRACT

In this paper, we aggregate, analyze, and exemplify several metrics for the information value. The metrics vary by absolute value, normalizations, or full probabilistic quantification. The normalization of the information value encompasses the division by (1) the expected and maximized utility of the base scenario (usually without additional information), (2) by the system performance, or (3) by the information acquirement costs. The latter can be aligned to the return over investment ratio to support and facilitate SHM system investment decision. With a case study on malevolent attacks to a generic iconic bridge, we exemplify the information value metrics and point to the application ranges and the implications of full distribution considerations.

INTRODUCTION

In the context of the Bayesian decision analysis, the information value constitutes the expected utility gain by information. In the last decade, the information value analysis has been significantly further developed and adapted to structural health monitoring (SHM) on built environment systems (see a review by [1]). For an information value and in alignment with the expected utility theorem, the expected utility gain by information should be higher than its cost. Whereas the absolute information value metric may be well understood by decision analysts, further metrics are required to adjust the information value quantification to different decision contexts.

We aggregate, analyze, and exemplify several metrics and relations of the information value. These metrics varied namely whether the metric is based on expected values or full distributions, whether it is absolute or normalized with the system's state utilities and consequences, or the information acquisition costs.

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The information value metrics are aggregated based on system performance, predicted action, and predicted information analyses. As a case study, an iconic bridge threatened by a malevolent attack with improvised explosive device is considered. The risk mitigation strategy modelled is the implementation of a surveillance system in combination with bridge temporary closure during times of threat indications. The paper closes with conclusions for the employment of different information value metrics.

VALUE OF INFORMATION ANALYSIS AND METRICS

Value of information analyses build upon decision analyses. Decision analyses are classified according to the information acquisition state (as originally formulated [2], [3]) and the action implementation state (as recently formulated [4]).

Figure 1 contains the basic information, action, system and utility modelling for a predicted information and predicted action (PIPA) decision analysis (DA), for a predicted action (PA) DA and the system performance (SP) analysis (A). The SP A is defined as the expectation of the system state utilities:

$$U_{SP} = E_{X_l}\left[U\left(X_l\right)\right]. \tag{1}$$

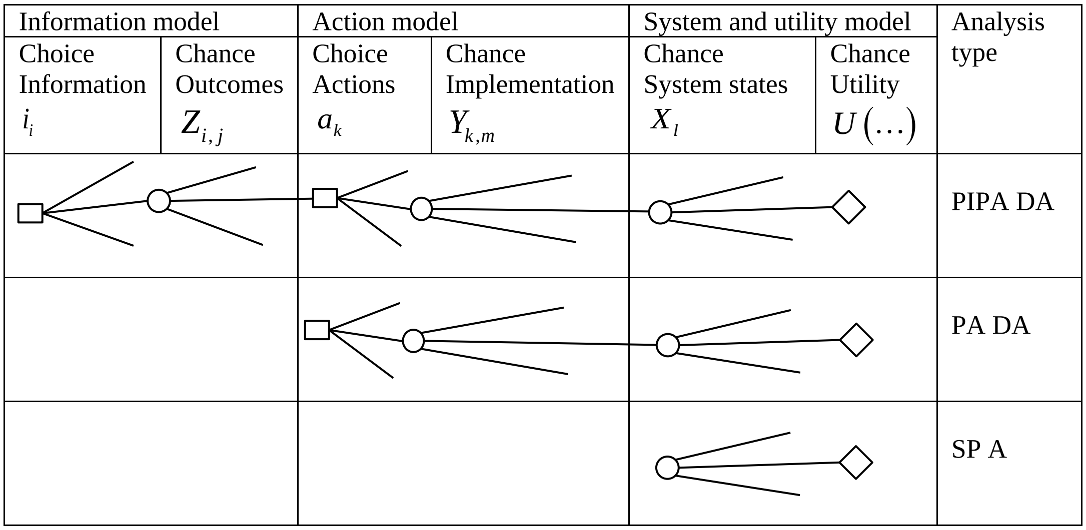

Figure 1: Basic decision analysis types for predicted decision analyses with decisions trees

The objective functions (2) and (3) for the PIPA and PA DA are written to maximize the expectation of utility, as per the expected utility theorem [5], while considering modifications based on information and actions. The actions can be distinguished into system state actions influencing the system state ($X_l\left(Y_{k,m},a_k\right)$) and utility actions influencing the system state utilities ($U\left(X_l,Y_{k,m},a_k\right)$) [6]. In accordance with the expected utility theorem, the decision analysis involves the expectations of both action and information costs. Here, probabilistic cost models for the information acquirement strategies $C(i_i)$ and the actions $C(a_k)$ are modelled.

$$U_{PIPA} = \max_{i_i, a_k} E_{X_l(Y_{k,m}, a_k)} \left[E_{Y_{k,m}} \left[E_{Z_{i,j}|X_l(Y_{k,m}, a_k)} \left[U(X_l, Y_{k,m}, a_k) - E[C(a_k)] \right] \right] - E[C(i_i)] \right] \quad (2)$$

$$U_{PA} = \max_{a_k} E_{Y_{k,m}} \left[E_{X_l(a_k, Y_{k,m})} \left[U(X_l, a_k, Y_{k,m}) - E[C(a_k)] \right] \right] \quad (3)$$

The value of information V constitutes the information induced difference of maximized and expected utilities between a base scenario and an enhancement scenario [4]. Here, we limit the scope to a base scenario without information and an enhancement scenario with predicted information (PI):

$$V_{PI} = U_{PIPA} - U_{PA}. \quad (4)$$

The above objective functions (2) and (3) are formulated so that the information value is quantified with the information costs; however, the information value quantification may exclude the information costs [2], [3] as per individual definitions.

Several metrics for the information value have been proposed in scientific literature. The metrics vary by absolute value, normalizations or full probabilistic quantification. A normalized information value $\overline{V}$ can be calculated with the expected and maximized utility of the base scenario (here $\overline{V}_{PI/U_{PA}}$, see Equation (5) and , and [4] for reference), with the system performance ($\overline{V}_{PI/U_{SP}}$, see Equation (6) and references [10] and [6]), or with the information acquirement costs ($\overline{V}_{PI/c(i)}$, see Equation (7) and reference [11]):

$$\overline{V}_{PI/U_{PA}} = \left(U_{PIPA} - U_{PA}\right) / U_{PA}, \quad (5)$$

$$\overline{V}_{PI/U_{SP}} = \left(U_{PIPA} - U_{PA}\right) / U_{SP}, \quad (6)$$

$$\overline{V}_{PI/c(i)} = \left(U_{PIPA} - U_{PA}\right) / c(i). \quad (7)$$

These metrics may be interpreted as information significance (see [10]) of the information value in relation to the risk analysis, to the PA DA and to the information costs.

It should further be noted that utility distributions, i.e., risk and expected cost distributions may be used for the information value quantification (Equations (4) to (7)) as performed for the risk reduction strategy significance and effectiveness quantification in [10].

CASE STUDY

As a case study, an iconic bridge with a value of €2.0 Billon or annual cost of €81 Million for a 100-year design life and an interest rate of 4% is considered. The risk mitigation strategy to be considered is the implementation of a surveillance system in combination with bridge temporary closure during times of high threats taking basis in [12] and [13]. The SP analysis constitutes a risk analysis and the failure probability $P\left(X_1 = F\right)$ is calculated with Equation (8) including the threat probability $P\left(T\right)$, the probability of a hazard given the threat $P\left(H \mid T\right)$, and the probability of collapse $P\left(F \mid H\right)$, see [14]:

$$P\left(F\right) = P\left(F \mid H\right) \cdot P\left(H \mid T\right) \cdot P\left(T\right). \tag{8}$$

The threat probability is defined as the probability that attackers work on an attack plot. As this probability is rather publicly unknown, threat probabilities between and 0.1 for an immediate threat and 0.0001 for a low threat probability are analyzed.

The probability of a hazard for the bridge given the threat $P\left(H \mid T\right)$ can be derived by the detonation performance of an IED. Although IED detonations are generally considered to be technically reliable [15], human errors significantly reduce the hazard probability. To account for a presumably skilled attack on an iconic bridge, we employ a triangular distribution characterized by a range between 0% and 25%, with a mode of 15%, see also [16]. The probability of a bridge collapse given a hazard $P\left(F \mid H\right)$ is high due to the likelihood of a large amount of explosive material in the IED, skilled attackers be able to design a collapse scenario, and the often low redundancy of structural systems in iconic bridges. The probabilistic model for the failure probability calculation is summarized in TABLE 1.

TABLE 1: PROBABILISTIC SYSTEM STATE, I.E., FAILURE SCENARIO, MODEL

Denotation	Probabilistic model
Threat probability	$P\left(T\right)$: ranges from 0.1 to 0.0001
Conditional hazard probability	$P\left(H \mid T\right) \sim Tr\left(0.0, 0.15, 0.25\right)$
Conditional progressive collapse probability	$P\left(F \mid H\right) = 90\%$

Tr: Triangular distribution

A bridge collapse results in property damage, reconstruction costs for the bridge and vehicles, as well as fatalities, indirect costs associated with its significance in the road network (traffic diversions, user delays, business losses), and social losses reflecting fear, anxiety, and potential impact on civil liberties. Following the discussion in [14], a Triangular consequence distribution normalized to the bridge value between 5 and 25 with a mode at 10 is modelled (TABLE 2).

A surveillance-based control strategy, along with bridge closure, can be employed. If surveillance detects an imminent threat, a temporary bridge closure is performed to allow time for the apprehension of suspects or the safe defusal of the

IED. The cost of bridge closure is determined by the bridge's importance in the traffic network and associated costs, see e.g., [24] and [33]. Modelling a 2-day closure for prevention and detection of explosive charges, the estimated cost amounts to 0.27% of the bridge value. Bridge closure mitigates the potential consequences of a collapse, reducing property damage, the number of vehicles affected, and related costs such as fatalities, reconstruction, and traffic diversion. The consequences of bridge failure given closure are lowered and a Uniform distribution of factors of the bridge value between 1.0 and 5.0 is chosen (TABLE 2). The bridge closure action is modeled without action implementation uncertainties for simplicity.

A uniform surveillance detection probability distribution with a range of 0.7 to 0.99 is chosen for the correct indications, i.e., the probability of a threat indication given a threat $P\left(Z_{2,T} \mid T\right)$ and the probability of no threat indication given no threat $P\left(\bar{Z}_{2,T} \mid \bar{T}\right)$ based on similar modelling in [14] and [10]:

$$\mathbf{P}\left(\mathbf{Z}_{2,T} \mid \mathbf{T}\right) = \begin{bmatrix} P\left(Z_{2,T} \mid T\right) & P\left(Z_{2,T} \mid \bar{T}\right) \\ P\left(\bar{Z}_{2,T} \mid T\right) & P\left(\bar{Z}_{2,T} \mid \bar{T}\right) \end{bmatrix}. \tag{9}$$

A pre-posterior Bayesian updating of the threat probability is performed with Equation (10):

$$P\left(T \mid \mathbf{Z}_{2,T}\right) \cdot P\left(\mathbf{Z}_{2,T}\right) = P\left(\mathbf{Z}_{2,T} \mid T\right) \cdot P\left(T\right). \tag{10}$$

TABLE 2: PROBABILISTIC INFORMATION, ACTION, COST, AND CONSEQUENCE MODELS

Nodes		Description	Probability		Cost	Consequence
X	X_1=F	System failure	$P(F)$		-	$C(F) \sim Tr(5.0,10.0,25.0)$
	X_2=S	Safe	$P(S)=1-P(F)$		0	-
a	a_0	Do nothing	-		-	-
	a_1	Close bridge	-		$c(a_1)=0.27\%$	$C(F(a_1)) \sim Uni(1.0,5.0)$
Z			Threat T	No Threat $\bar{T}$		
	Z_T	Threat indication	$P(Z_T \mid T) \sim U(0.7,0.99)$	$1-P(Z_T \mid \bar{T})$	$C(i_{Surv}) \sim Uni(10^{-4};10^{-3})$	-
	$\bar{Z}_T$	No threat indication	$1-P(\bar{Z}_T \mid T)$	$P(\bar{Z}_T \mid \bar{T}) \sim U(0.7,0.99)$		-

Uni: Uniform distribution, *Tr*: Triangular distribution

The surveillance cost $C\left(i_{Surv}\right)$ is modelled with a Uniform distribution in a range between 1.0E-4 and 1.0E-3. The cost distribution is fully correlated with the correct indications, i.e., the surveillance system with a high reliability has high costs.

Figure 2 depicts the value of predicted information. There is a positive surveillance information value in the threat probability range of 1.0E-3 and 9.0E-3 with the highest information value for 3.0E-3. The information value including the cost of information drops to the information costs outside this range – as per definition (Equation (2)).

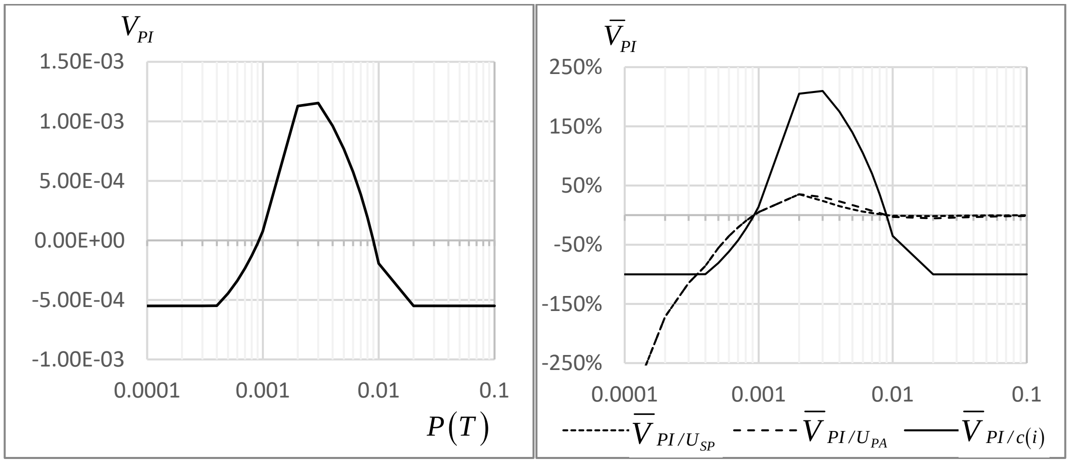

Figure 2: Value of surveillance information (left) and value of surveillance information in different metrics (right)

The normalized information value with the SP A and the PA DA results, in (Figure 2, right, dashed, and dotted lines) reveals the risk and expected cost reduction as a fraction of the normalizing risk (SP A) and the closure action adapted risk (PA DA) up to 35%. The information value normalized with SP A and PA DA results coincide in the diagram apart from the range 3.0E-3 to 9.0E-3. The reason lies in the fact the information value is provided in this range by the change of action implementation (PA DA) to doing nothing (PIPA DA) for a no threat indication. For the do-nothing action in the PA DA, the risks and expected costs coincide for the PA DA and the SP A.

Surveillance information value normalization by surveillance costs (Figure 2, right, continuous line) may serve as a basis for an investment decision of the surveillance system. The value of the surveillance system information is composed of a risk reduction and an expected cost reduction up to 210% of the expected cost of surveillance for a threat probability of 3.0E-3. Full probabilistic utility, i.e., risk and expected cost distributions may be derived for a specific threat probability (Figure 3) and the information value may be computed with Equations (4) and (7). From the information value distributions, exceedance probability distributions ($P_{Ex}\left(\bar{V}_{PI}\right)=1-F\left(\bar{V}_{PI}\right)$) are derived. For both normalizations, the probability of a positive information value is about 81%. With a probability of 50%, an approx. 30%

risk and expected cost reduction in regard to the PA DA and an approx. 200% reduction in respect to information costs is achieved. The different order of fractions is due to the fact that the surveillance system costs are significantly lower than the risk from the PA DA.

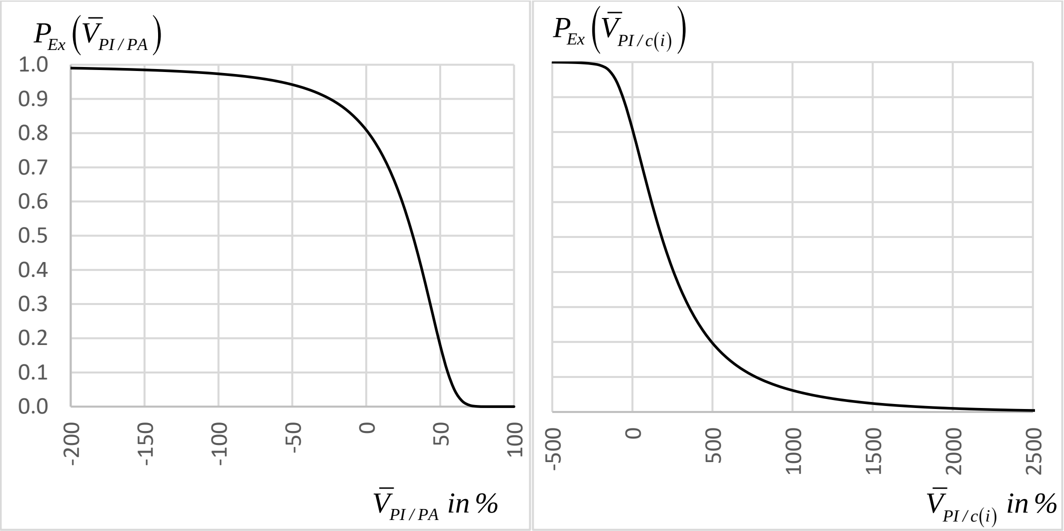

Figure 3: Exceedance probabilities of normalised information value for a threat probability of 3.0E-3

The exceedance probability distribution for the information value is closely related to effectiveness measure as proposed in [10]. The probability of effectiveness is defined as the zero exceedance probability of the difference between the risk and the risk reduction strategy cost distributions [10].

SUMMARY AND CONCLUSIONS

Absolute and normalized information value metrics are aggregated, analyzed, and exemplified. The metrics are the absolute information value and the information value normalized with the expected value and the distributions of the risk, the action adapted utility and the information cost. The metrics add knowledge about the information significance for different decision makers namely (1) for owners responsible for the risk management of an infrastructure by relation to the risk, (2) for risk managers who want to improve their risk management strategies with information by relation to the action adapted risk and (3) for the SHM industry by a risk reduction encompassing return over investment ratio. The quantification of the information value exceedance probability distribution complements information value analyses towards decision implementation (see also [10] for effectiveness quantification of risk reduction strategies).

The information value normalized by the information acquisition costs reveals a very high value despite potentially high costs for SHM system research, development, design, installation, operation, and maintenance as well as potential replacement.

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